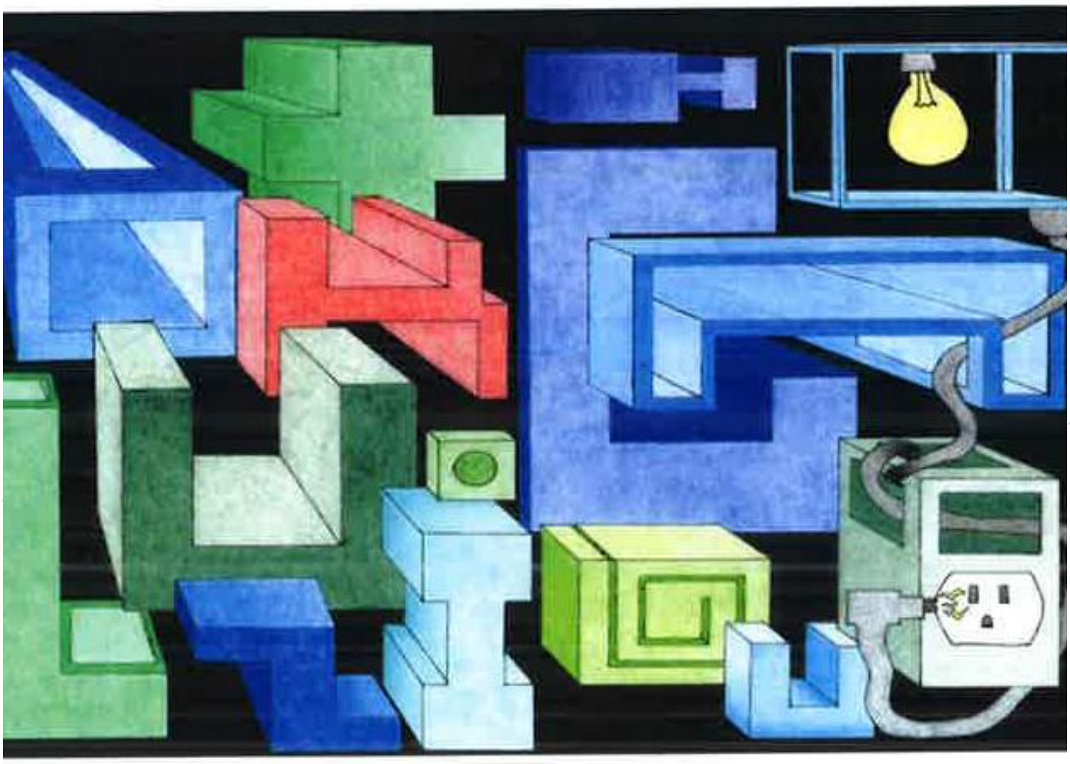
DEPARTMENT OF EDUCATION

GRADE 9

MATHEMATICS

UNIT 5



DESIGN IN 2D AND 3D GEOMETRY (1)

Published by:

FLEXIBLE OPEN AND DISTANCE EDUCATION
PRIVATE MAIL BAG, P.O. WAIGANI, NCD
FOR DEPARTMENT OF EDUCATION
PAPUA NEW GUINEA

GRADE 9

MATHEMATICS

UNIT 5

DESIGN IN 2D AND 3D GEOMETRY (1)

Acknowledgements

We acknowledge the contributions of all Secondary teachers and people who in one way or another helped to develop this Course.

Special thanks is given to the Staff of Mathematics Department of FODE who played active role in coordinating writing workshops, outsourcing lesson writing and editing processes, involving selected teachers from NCD.

We also acknowledge the professional guidance provided by the Curriculum Development and Assessment Division throughout the processes of writing and, the services given by the members of the Mathematics Subject Review and Academic Committees.

MR. DEMAS TONGOGO
Principal-FODE

Flexible Open and Distance Education
Papua New Guinea

Published in 2016

ISBN: 978-9980-87-738-3

National Library Services of Papua New Guinea

Written by: Luzviminda B. Fernandez

Printed by the Flexible, Open and Distance Education

CONTENTS

SECRETARY'S MESSAGE

Achieving a better future by individual students and their families, communities or the nation as a whole, depends on the kind of curriculum and the way it is delivered.

This course is part and parcel of the new reformed curriculum. The learning outcomes are student-centered with demonstrations and activities that can be assessed.

It maintains the rationale, goals, aims and principles of the national curriculum and identifies the knowledge, skills, attitudes and values that students should achieve.

This is a provision by Flexible, Open and Distance Education as an alternative pathway of formal education.

The course promotes Papua New Guinea values and beliefs which are found in our Constitution and Government Policies. It is developed in line with the National Education Plans and addresses an increase in the number of school leavers as a result of lack of access to secondary and higher educational institutions.

Flexible, Open and Distance Education curriculum is guided by the Department of Education's Mission which is fivefold:

- to facilitate and promote the integral development of every individual
- to develop and encourage an education system that satisfies the requirements of Papua New Guinea and its people
- to establish, preserve and improve standards of education throughout Papua New Guinea
- to make the benefits of such education available as widely as possible to all of the people
- to make the education accessible to the poor and physically, mentally and socially handicapped as well as to those who are educationally disadvantaged.

The college is enhanced through this course to provide alternative and comparable pathways for students and adults to complete their education through a one system, two pathways and same outcomes.

It is our vision that Papua New Guineans' harness all appropriate and affordable technologies to pursue this program.

I commend all the teachers, curriculum writers and instructional designers who have contributed towards the development of this course.

....

DR. UKE KOMBRA, PhD

Acting Secretary for Education

UNIT 5: DESIGN IN 2D AND 3D GEOMETRY (1)

Dear student,

This is the Fifth unit of the Grade 9 Mathematics Course.
It is based on the NDOE Upper Primary Mathematics Syllabus and Curriculum framework for Grade 9.

This unit consists of four Topics:

Topic 1: Points, Lines, Planes and Angles
Topic 2: Polygons
Topic 3: Area
Topic 4: Surface Area and Volume

In Topic 1- **Points, Lines, Planes and Angles**- You will make models of solids to help you learn more about planes, lines and angles.

In Topic 2- **Polygons**- You will revise what you already know and learn more about polygons.

In Topic 3- **Area**- You will revise area of a rectangle, triangle and rhombus. You will also learn to calculate the area of a trapezium, parallelogram and composite shapes.

In Topic 4- **Surface Area and Volume**- You will learn to calculate the surface area of prisms including volume of cylinders. You will also learn to calculate the volume of cones, pyramids and spheres..

You will find that each lesson has reading materials to study, worked examples to help you, and a Practice Exercise for you to complete. The answers to the practice exercises are given at the end of each topic.

All the lessons are written in simple language with comic characters to guide you and many worked examples to help you. The practice exercises are graded to help you to learn the process of working out problems.

We hope you enjoy studying this book.

Mathematics Department
FODE

STUDY GUIDE

Follow the steps below and work through the lessons.

Step 1: Start with Topic 1 and work through it in order.

Step 2**:** When you complete Lesson 1, do Practice Exercise 1.

Step 3: After you have completed each exercise, correct your work. The answers are given at the end of each Topic.

Step 4: Then revise the lesson and correct your mistakes if any.

Step 5: When you have completed all these steps, tick the check-box for Lesson 1 on the Content page like shown:

✔ Lesson 1: Points, Lines and Angles

Then go on to the next lesson. Repeat this process, until you complete all the lessons in a Topic. When this is done, revise using the review section.

As you complete each lesson tick the box for that lesson, on the Contents page, like this ✔. This helps you to check on your progress.

Assignment: Topic Tests and Unit Test.

When you have completed all the lessons in a Topic do the Topic Test for that Topic in your Assignment Book. The Unit book tells you when to do this.

When you have completed all Topic Tests for a Unit, revise well and do the Unit Test. The Assignment Book tells you when to do the Unit Test.

Your Distance Teacher will mark the Topic Tests and the Unit Test in each Assignment. The marks you score in each Assignment will count towards your final result. If you score less than 50%, you will repeat the Assignment.

Remember, if you score less than 50% in three Assignments, you will not be allowed to continue. So, work carefully and ensure that you pass all the Assignments.

TOPIC 1

POINTS, LINES, ANGLES AND PLANES

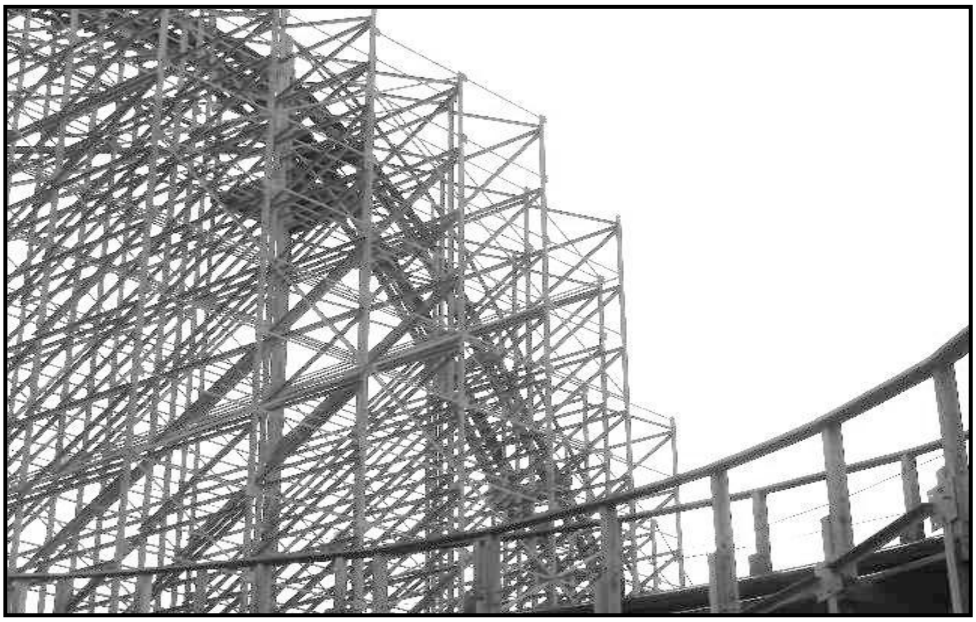
TOPIC 1: POINTS, LINES, ANGLES AND PLANES

Introduction

Geometry is about points, lines and figures which can be drawn with them. A very important skill is to be able to interpret a question and draw a diagram**.**

Point, line, angles and plane are the building blocks of design. From these elements, designers create images, icons, textures, patterns, diagrams, animations, and typographic systems.

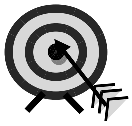
The picture below is only one of the examples of real life application of points, lines and angles.



This topic reviews the basic terms and concepts in geometry and provides further lessons to help you develop your understanding of geometry and its applications to solving problems in real life.

Lesson 1: Points, Lines, Angles and Planes

In Grade 8, you have learnt the most basic ideas in Geometry.



In this lesson, you will:

- define and name points
- define and name planes
- define and name lines
- draw a line
- define and name angles .

Geometry is about the shape and size of things. It is the study of points, lines, angles, shapes, their relationships, and their properties. The most basic ideas of geometry (and other mathematical ideas) are point , line, plane and angle.

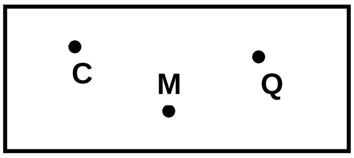
This topic starts with an understanding of these basic geometric ideas.

Let us look at them one by one.

Point

A point is the most fundamental geometric magnitude. It has no dimension. It is a fixed location in space with no definite form size or shape. It is represented by a dot like a full stop.

The diagram below illustrates point **C**, point **M**, and point **Q**.

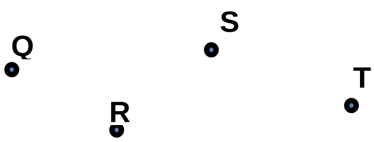


Kinds of Points

Collinear points is a set of points that lie in a straight line

We say that "point O is collinear with points M, N and P". Or put another way, "the points M, N, O and P are collinear".

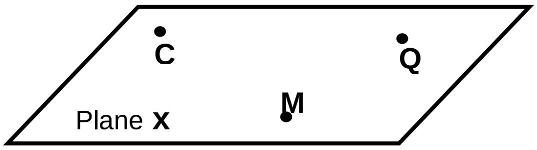
Non-collinear points are set of points that do not lie on the same line.



Notice that there is no line on which all of the points lie.

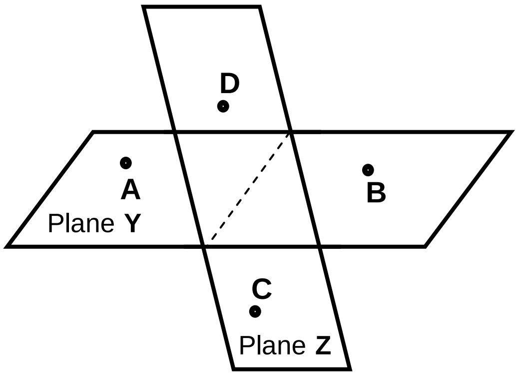
We say that "point **S** is non-collinear with points **Q**, **R** and **T**". Or put another way, "the points **Q**, **R**, **S** and **T** are non-collinear".

Coplanar points are set of points that lie on the same plane.



All points **C**, **M** and **Q** the Plane **x** are coplanar.

Non-coplanar points are set of points that do not lie on the same plane.

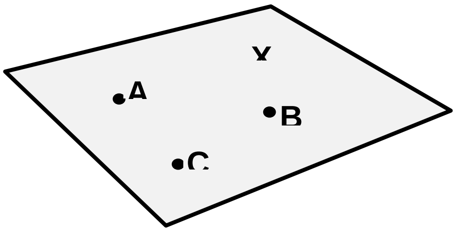


Points **A**, **B**, **C** and **D** lie on two different planes so, they are non-coplanar points.

Plane

A plane is a flat surface that extends infinitely in all directions. It has two dimensions; its length and its width.

A plane can be named either by a single capital letter or by naming at least three non-collinear points within the plane. (See diagram on the next page.)

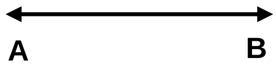


This is Plane **X** or Plane **ABC**.

Line

A line is a geometric magnitude formed by a set of points moving indefinitely in any direction along a straightedge. It has one dimension, its length.

A line is named by any two points on it. The symbol $\leftrightarrow$ written on top of two capital letters is used to denote a line. A line may also be named by one small letter associated with it.



This is line AB. Denoted by $\overleftrightarrow{AB}$

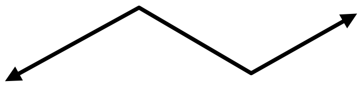
This is line m. Denoted by $\overleftrightarrow{m}$

Kinds of Line

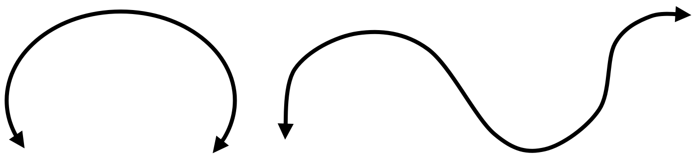
1. **Straight Line** - is a line extending infinitely in opposite direction following a straight edge.

This is line AB. It is written as $\overleftrightarrow{AB}$

2. **Broken Line** – is a line made up of successive straight line segments but they themselves do not form a straight line.



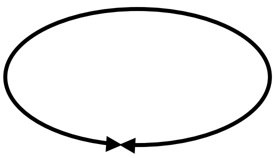
3. **Curved Line** – is a line wherein no part of it is straight.



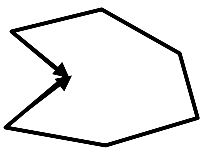
When the endpoints of a line meet, a **closed figure** is formed.

A **closed curved line** is formed when the endpoints of a curved line meet.

A **closed broken line** is formed when the endpoints of a broken line meet.



Closed curved line



Closed broken line

The concept of lines is straightforward, but much of geometry is concerned with portions of lines. Some of those portions such as **Segments, Midpoints, and Rays** are so special that they have their own names and symbols.

Line segment

A **line segment** is a part of a **line** that is bounded by two distinct end points, and contains every point on the **line** between its end points.

A line segment is a connected piece of a line. It has two endpoints and is named by its endpoints. Sometimes, a bar line or symbol (—) is written on top of two letters to denote the segment.

In the diagram, four points **A**, **B**, **C**, and **D** create several line segments such as:

Line segment AC or $\overline{\mathbf{AC}}$

Line segment CD or $\overline{\mathbf{CD}}$

Line segment DB or $\overline{\mathbf{DB}}$

Line segment AD or $\overline{\mathbf{AD}}$

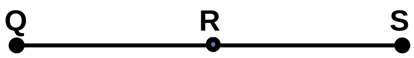
Line segment CB or $\overline{\mathbf{CB}}$

Line segment AB or $\overline{\mathbf{AB}}$

Note that $\overline{\mathbf{AC}}$, $\overline{\mathbf{CD}}$, $\overline{\mathbf{DB}}$, $\overline{\mathbf{AD}}$ and $\overline{\mathbf{CB}}$ are a parts of $\overline{\mathbf{AB}}$.

Midpoint

A **midpoint** of a line segment is the halfway point, or the point equidistant from the endpoints



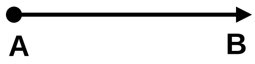
The Midpoint of a line segment **QS** is Point **R**

Point **R** is the midpoint of $\overline{\mathbf{QS}}$ since $\overline{\mathbf{QR}} = \overline{\mathbf{RS}}$ or because $\overline{\mathbf{QR}} = ½\ \overline{\mathbf{QS}}$ or $\overline{\mathbf{RS}} = ½\ \overline{\mathbf{QS}}$
A line segment has exactly one midpoint.

Ray

A **ray** is also a piece of a line, except that it has only one endpoint and continues forever in one direction. It could be thought of as a half-line with an endpoint.

A ray is named by the letter of its endpoint and any other point on the ray. The symbol ●⟶ written on top of the two letters is used to denote that ray. This is ray **AB.**

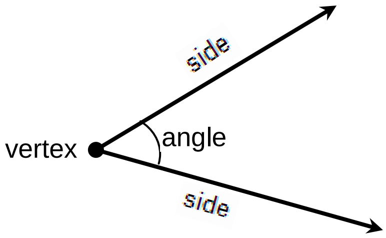


Ray **AB**

To denote a ray the non-arrow part of the ray is on the starting point. This is written as $\overrightarrow{AB}$..

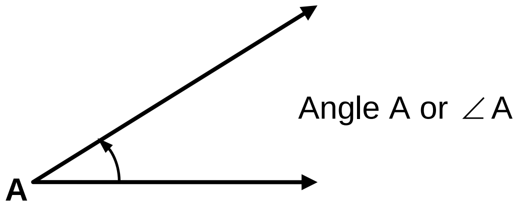
Angles

An **angle** is a plane figure formed by two rays or lines that diverge or extend from a common endpoint and do not lie on a straight line. The common endpoint is the **vertex** and the two rays are the **sides.**

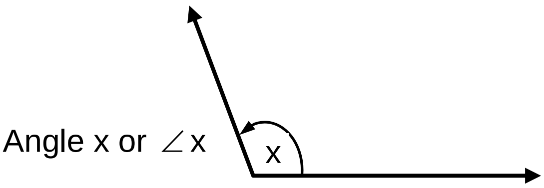


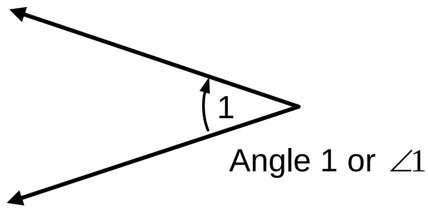
Angles are identified or named in the following three ways:

a) By reading the capital letter at the vertex

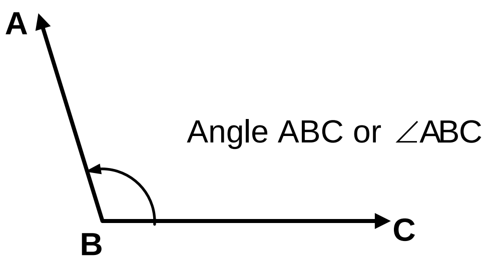


b) By reading the inside letter or numeral





c) By reading the three letters associated with the sides and the vertex.



The symbol to denote or stand for the word **angle** is '$\angle$'. The angles above may be $\angle A$, $\angle x$, $\angle 1$ or $\angle ABC$ with the letter at the vertex in the middle.

In $\angle ABC$, the sides are ray BC and ray BA. The first letter is the letter at its endpoint.

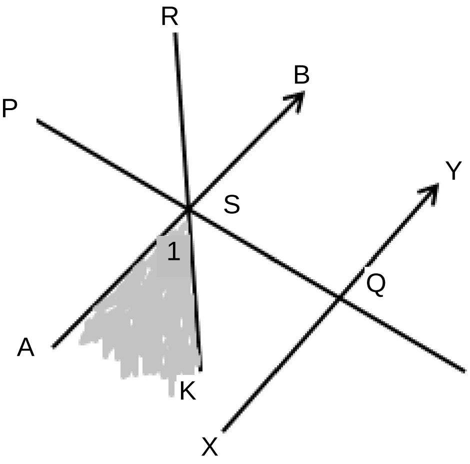
$\bullet\!\!\longrightarrow$ is the symbol for **ray**.

So, ray BC can be written as $\overrightarrow{BC}$ and ray BA can be written as $\overrightarrow{BA}$.

NOW DO PRACTICE EXERCISE 1

Practice Exercise 1

1. Refer to the diagram to answer the following questions.



a) Are points A, S and Q collinear? ____________

b) At which point do lines PQ and RK intersect? ____________

c) Which two lines are parallel? ____________

d) What is the vertex of $\angle$SQX? ____________

e) Name the shaded angle in two different ways. ____________

f) Name two points collinear with B. ____________

---

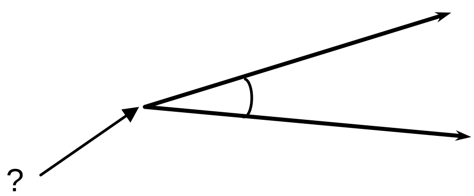
2. What word best describe each of the following.

a) The point where the arms of an angle meet. ____________

b) Points which lie on the same straight line. ____________

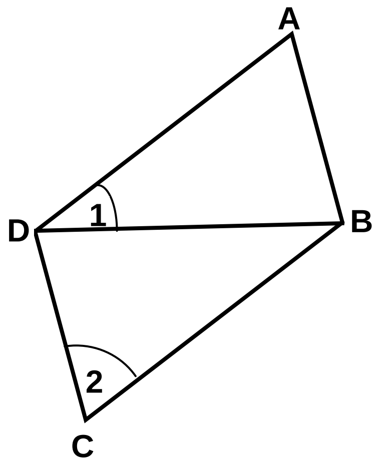
c) The halfway point of a line segment. ____________

Refer to the diagram below to answer question 3.



3. The arrow points to the angle's ___________.

Refer to the diagram below to answer question 3.



4. Using the three letters on the shape that define the angle,

   (a) write the name of $\angle 1$ _______________

   (b) and $\angle 2$. _______________

Refer to the figure below to answer questions 5 and 6.

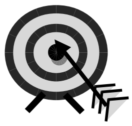
5. How many segments can be named from the figure below?

6. Name and list the segments that can be formed on the figure.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 2: Types of Angles

In the previous lesson, you learnt what angles are and how to name angles.

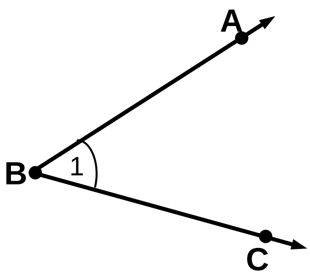


In this lesson, you will:

- revise the meaning of angles
- identify and name correctly the different types of angles
- define what is a protractor
- measure angles using a protractor
- draw angles using a protractor.

First, let us revise what you have learnt earlier about angles. Here again is the meaning of angle.

An angle is formed when two lines or rays meet at a common endpoint.



Points **A**, **B** and **C** are points on the angle.
Point **B** is the endpoint where they meet and it is called the **vertex** of the angle.

AB and **BC** are the two lines or rays that formed the angle. They are called the **sides** or **arms** of the angle.

The angle can be named in a number of ways:

- by the three capital letters associated with the sides or arms: as $\angle$**ABC** (counter-clockwise) or $\angle$**CBA** (clockwise) where vertex **B** is always at the middle**.**
- by the letter written on the vertex: as $\angle$**B**
- by the small numeral written inside the angle: as $\angle$**1**

Now that you know how to name angles, you are now going to look at the different types or classification of angles determined by their measures.

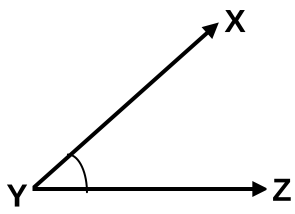
Angles are measured in **degrees**. We use the small circle (**°**) as a symbol to mean degrees.

Here are the types of angles.

Zero angle is formed when two rays diverge (move away) from the same point and then coincide.

$\angle\alpha$ is a zero angle. There is no rotation made by the rays. $\angle\alpha = 0^\circ$

Acute angle is any angle which measures less than 90º.

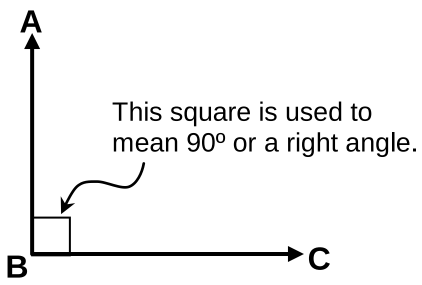


$\angle$**XYZ** is an acute angle. $\angle$**XYZ** $> 0^\circ$ but $< 90^\circ$

Any angle that forms a sharp corner, such as the corner of a table, corner of a blackboard or a corner of a desk is called a **right angle**.

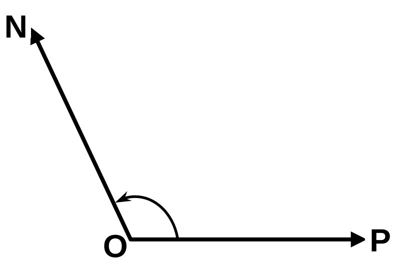
Right angle is any angle which measures exactly 90º.

A right angle looks like this:



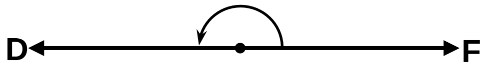
$\angle$**ABC** is a right angle. $\angle$**ABC** = 90

Obtuse angle is any angle which measures greater than 90º but less than 180º.



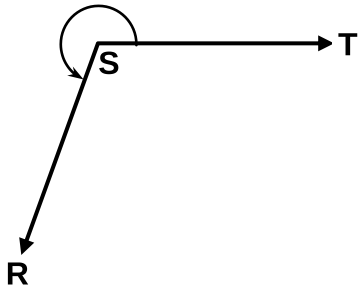
$\angle$**NOP** is an obtuse angle. $\angle$**NOP** $>90^\circ$ but $< 180^\circ$

Straight angle is an angle which measures exactly 180º.



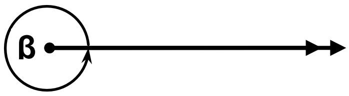
$\angle DEF$ is a straight angle. $\angle DEF = 180^{\circ}$

Reflex angle is any angle which measures greater than 180º but less than 360º.



$\angle RST$ is a reflex angle. $\angle RST > 180^{\circ}$ but $< 360^{\circ}$

Revolution or Full Rotation is any angle which measures exactly 360º.



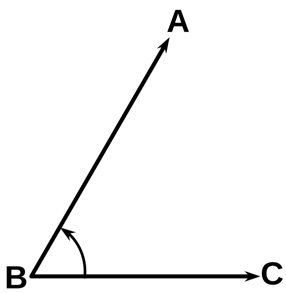
$\angle\beta$ is one revolution or full rotation. $\angle\beta = 360^{\circ}$.

Knowing the different types of angles, let us now learn how to measure and draw angles.

Measuring Angles

The size of an angle is usually measured in degrees (°). Think of a circle which is divided into 360 equal sectors. Each sector has a small angle at the centre of the circle – the size of such an angle is 1 degree (written: 1°). In other words, the size of an angle is a measure of how wide the angle is at the vertex bound by the two sides.

Given an angle, say $\angle ABC$.



How do we measure the size of the angle?

We can measure angles using an instrument known as a protractor.

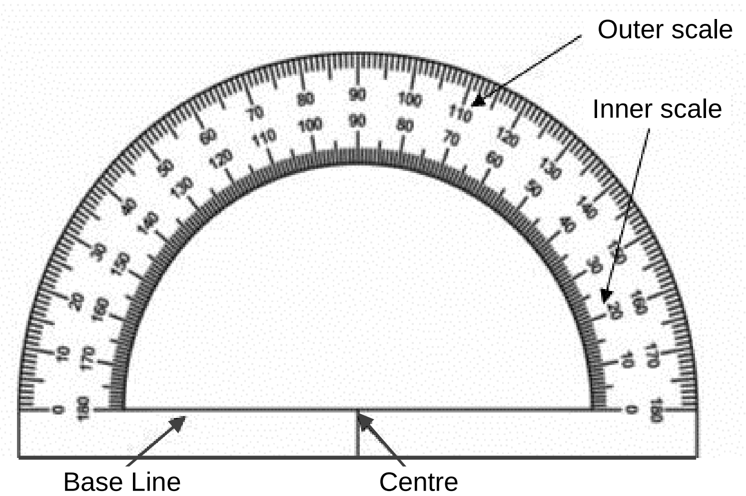
A **protractor** is an instrument used to measure the size of angles.

The protractor shown below has two scales:

The outer scale starts from 0˚ to180˚ going clockwise.

The inner scale starts from 0˚ to 180˚ going anti-clockwise.

A Protractor



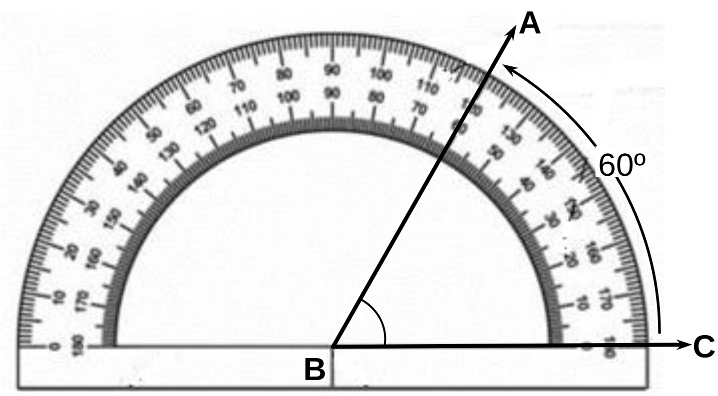
How to measure an angle using a protractor?

To measure $\angle$**ABC**, we need to:

Step 1: Place the centre point of the protractor on the vertex **B**.

Step 2: Adjust the base line of the protractor so that it is aligned with the line **BC**.

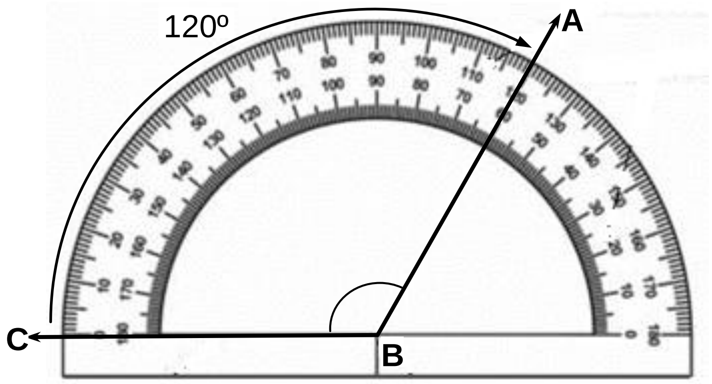
Step 3: Read the value of $\angle$**ABC**, from the inner scale.



The measure (m)of $\angle$**ABC** is 60º.

If the angle is facing the other way, we could use the outer scale to measure.

See diagram on the next page.



The measure (m) of $\angle$**ABC** is 120º.

Now let us have examples on how to draw angles using a protractor.

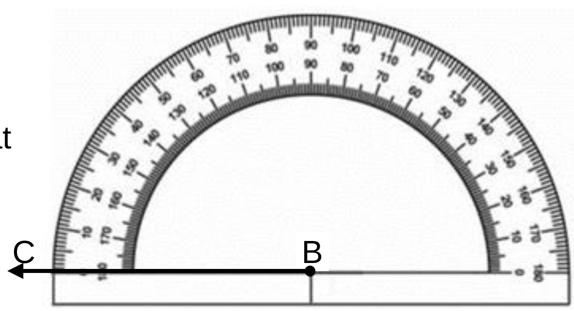
You will need a protractor and a pencil.

Drawing angles less than 180º.

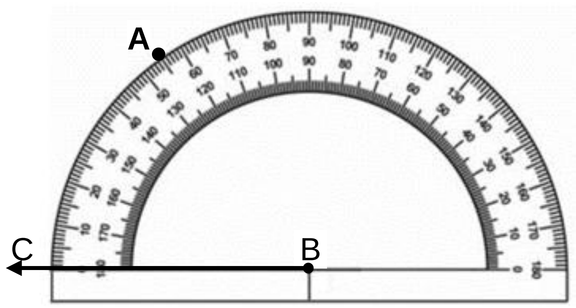
Example 1 Draw an angle **ABC**, which is 55º.

Step 1: Draw a ray **BC** and mark **B** as the vertex.

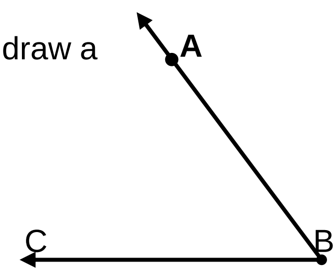
Step 2: Place the centre of the protractor at the vertex **B** and adjust the base line to be aligned to **BC**.



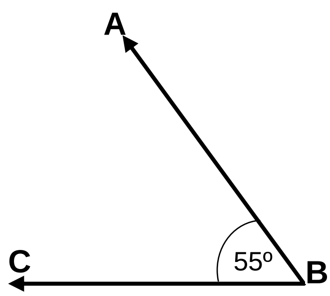
Step 3: Find 55º on the outer scale of the protractor and mark that point as Point **A**.



Step 4: Remove the protractor and draw a ray from point B to point A.



Step 5: Label the angle **ABC**.



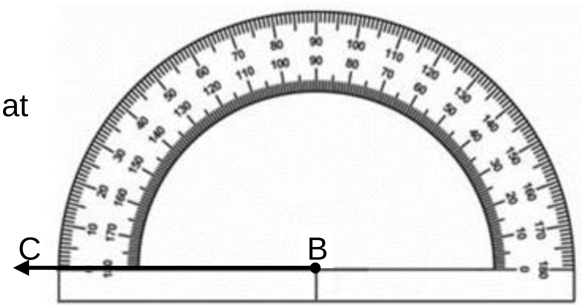
Drawing an angle greater than 180º but less than 360º (reflex angle)

Example 2 Draw an angle ABC which is 250º.

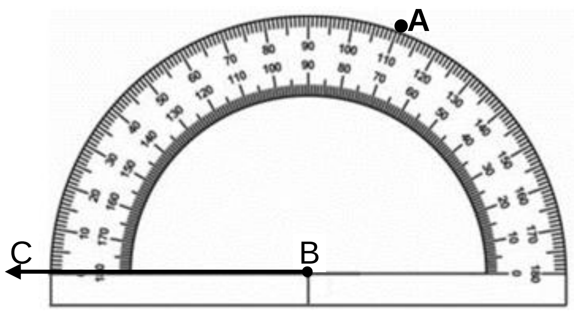
To draw the reflex angle, 250º, we first draw the angle obtained by subtracting 250˚ from 360˚ which is 360º – 250º = 110º.

Step 1: Draw a ray **BC** and mark **B** as the vertex.

Step 2: Place the centre of the protractor at the vertex **B** and adjust the base line to be aligned to **BC**.

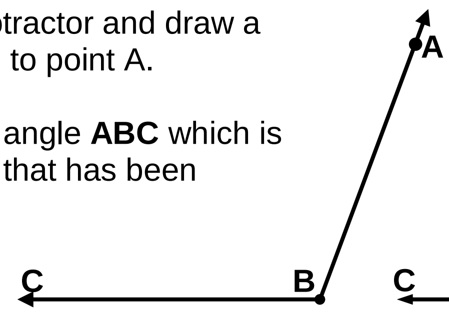


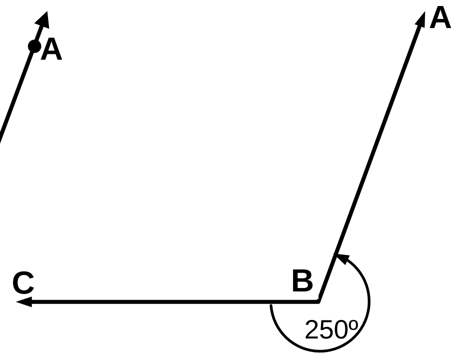
Step 3: Find 110º on the outer scale of the protractor and mark that point as Point **A**.



Step 4: Remove the protractor and draw a ray from point B to point A.

Step 5: Label the reflex angle **ABC** which is outside the one that has been drawn.





To draw a reflex angle (i.e. angle greater than 180º but less than 360º), we need to:

- Subtract the reflex angle from 360º.
- Draw the resulting angle as described above.
- Mark the required angle which is outside the one that has been drawn.

Remember:

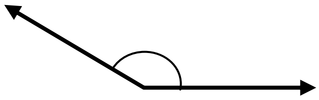
Protractors usually have two sets of numbers going in opposite directions. Be careful which one you use!

NOW DO PRACTICE EXERCISE 2

Practice Exercise 2

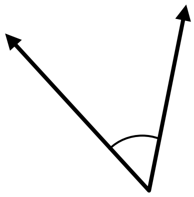
1. Classify each of the following angles as acute, right, obtuse or reflex.

(a)



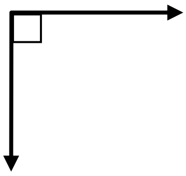
____

(b)



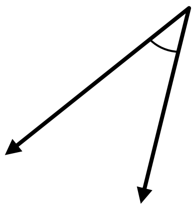
____

(c)



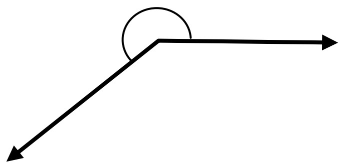
____

(d)



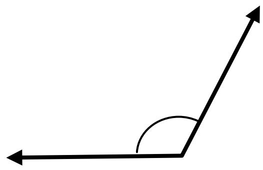
____

(e)



____

(f)



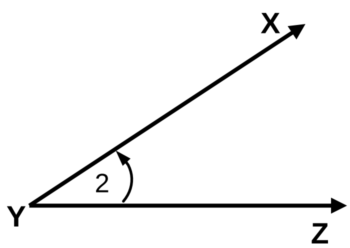
____

2. What type of angle is each of the following:

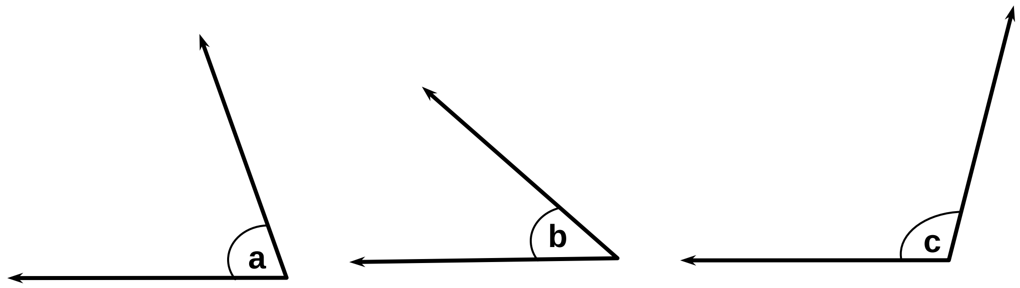
(a) 23° ________ (b) 123° ________ (c) 360° ________ (d) 180° ________

(e) 285° ________ (f) 90° ________ (g) 320° ________ (h) 4° ________

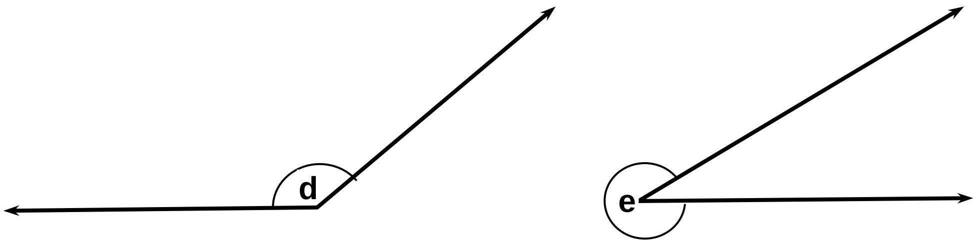
3. Name the angle in three different ways.



4. Using your protractor, find the measure in degrees of the following drawn angles.



m∠a = _______ m∠b = _______ m∠c = _______



m∠d = _______ m∠e = _______

---

5. Draw the following angle using your protractor.

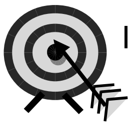
(a) ∠CAT which measures 270º

(b) ∠DOG which measures 75º

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 3: Complementary and Supplementary Angles

In the previous lesson you learnt the different types of angles according to their size. You also learnt to measure and draw angles using the protractor.

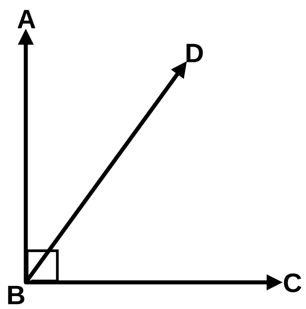


In this lesson, you will:

- define complementary and supplementary angles
- find the complement and supplement of angles
- solve problems involving complementary and supplementary angles

Some angles can be classified according to their positions or measurements in relation to other angles. We will examine two types of angles based on the sum of their respective measures: complementary and supplementary angles.

You learnt how to measure angles in the last lesson. Now, using your protractor, measure $\angle ABD$ and $\angle DBC$ below.



What is the total measure of the two angles?

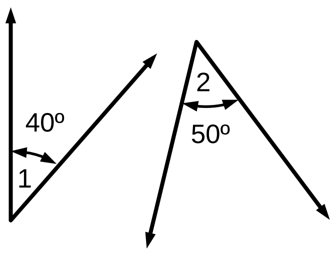
$\angle ABD + \angle DBC = 90$

These two angles are **Complementary Angles** because they add up to 90º.

$\angle ABD$ and $\angle DBC$ are two angles with a common side and a common vertex . Together they formed a right angle. They are called **adjacent complementary angles.**

But angles do not need to be adjacent to be complementary angles.

Here are two angles.



$\angle 1$ and $\angle 2$ are complementary angles since

$\angle 1 + \angle 2 = 40^\circ + 50^\circ = 90^\circ$

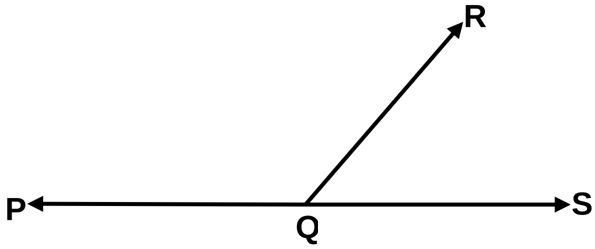
We say $\angle 1$ is the complement of $\angle 2$, and $\angle 2$ is the complement of $\angle 1$. Together they form a right angle.

If two angles add up to 90º, then they 'complement' each other.

Complement comes from Latin **completum** meaning "completed" ... because the right angle is thought of as being a complete angle.

Now look at the angles below.

$\angle PQR$ and $\angle RQS$ form a straight line and add up to 180º.

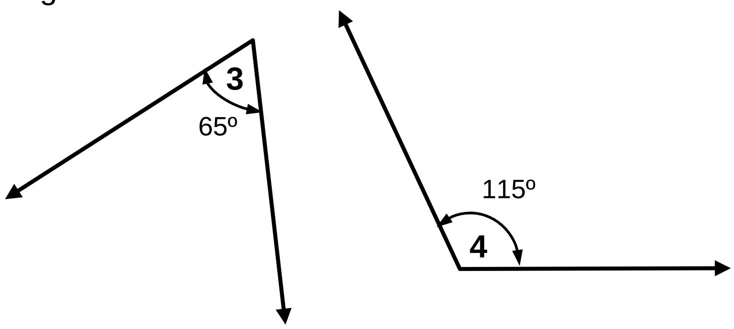


$\angle PQR + \angle RQS = 180º$

$\angle PQR$ and $\angle RQS$ share a common vertex and a common side. Together they form a straight angle.. They are called **adjacent supplementary angles**.

But angles do not need to be adjacent to be supplementary angles.

Here are two angles.



$\angle 3$ and $\angle 4$ are supplementary angles since $\angle 3 + \angle 4 = 65º + 115º = 180º$

We say $\angle 3$ is the supplement of $\angle 4$, and $\angle 4$ is the supplement of $\angle 3$. Together they form a straight angle.

If two angles add up to 180º, then they 'supplement' each other.

Supplement comes from Latin **supplere**, to complete or "supply" what is needed.

In summary we can say the following properties of angle pairs below:

- Complementary angles are any two angles whose sum is 90º and make a right angle.
- Supplementary angles are any two angles whose sum is 180º and make a straight angle.

Knowing these properties, you can use the idea of complementary and supplementary angles to help you solve problems.

We can work out the size of an unknown angle if we know the size of its complement or its supplement angle.

Go to the next page for the example problems.

These examples require your skills learnt in solving equations to solve for the unknown angle or angles.

Example 1

If two angles are complementary and one of them is 77°, what is the size of the other angle?

Solution: Complementary angles add to 90^o.

If we let x = the other angle

Then we have: $x + 77^o = 90^o$

Solve for x: $x = 90^o – 77^o$

$x = 13^o$

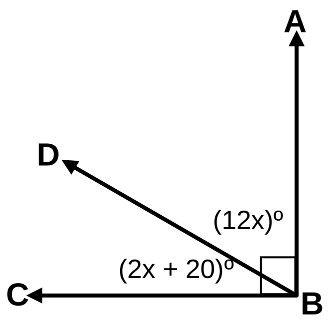
Check: $13^o + 77^o = 90^o$ ⟵ Substitute 13^o in place of x.

$90^o = 90^o$

Therefore, the size of the other angle is 13^o.

Example 2

Find the measures of the angles in diagram below, given that $\angle ABC$ is a right angle.



Solution:

Since $\angle ABC$ is a right angle, the two angles must be complementary i.e. they must add to 90°.

So, we have $(2x + 20)^o + 12x = 90^o$

$14x + 20^o = 90^o$

$14x = 70^o$

$x = 5^o$

Hence, $2(x) + 20 = 2(5) + 20 =$ **30^o** and $12(x) = 12(5) =$ **60^o**.

Check: $30^o + 60^o = 90^o$

$90^o = 90^o$

The two angles therefore, measure 30^o and 60^o.

Example 3

Two angles are supplementary and one of them is 127^o. What is the size of the other angle?

Solution: Supplementary angles add to 180^o.

If we let x = the other angle

Then we have: $x + 127^o = 180^o$

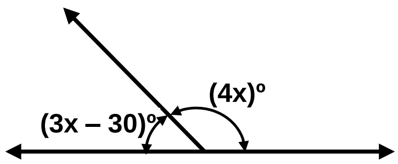
Solve for x: $x = 180^o - 127^o$

$x = 53^o$

Therefore, the size of the other angle is 53^o.

Example 4

Find the measure of each marked angle in the diagram.



Solution:

The measures of the marked angles must add to 180^0 since together they form a straight angle.

$(3x - 30) + 4x = 180$

$7x - 30 = 180$ Combine like terms

$7x - 30 + 30 = 180 + 30$ Add 30

$7x = 210$

$x = 30$ Divide by 7

Hence, one angle measures $3x - 30 = 3(30) - 30 = 60^o$. The other has $4(30) = 120^o$.

Check: $60^o + 120^o = 180^o$

The two angles therefore, measures are 60^o and 120^o.

NOW DO PRACTICE EXERCISE 3

Practice Exercise 3

A. Multiple choice

1. An angle measures 51^o. What is the measure of its supplement angle?

   (a) 129^o (b) 39^o

   (c) 309^o (d) 120^o

2. Find the measure of the angle complementary to the given angle, $y = 15^o$.

   (a) 75^o (b) 165^o

   (c) 90^o (d) 35^o

3. An angle measuring 32 degrees has a complement that measures (2x - 16) degrees. What is the value of x?

   (a) 82 (b) 37

   (c) 172 (d) 33

4. Two angles whose measures have a sum of 90 degrees.

   (a) Complementary Angles
   (b) Right Angles
   (c) Obtuse Angles
   (d) Supplementary Angles

5. Find the measure of an angle supplementary to the given angle, $m = 125^0$

   (a) 90^0 (b) 25^0

   (c) 65^0 (d) 55^0

B. Solve the following.

1. If one part of a right angle is 25^0, how big is the complementary angle?

---

2. If one part of a straight angle is 132^o, how big is the supplementary angle?

3. Two angles are supplementary and one of them is three times as big as the other. What is the size of the smaller of the two angles?

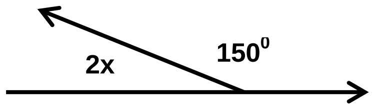
---

4. Two angles are supplementary and they are in the ratio 3 : 7. What is the size of the larger of the two angles?

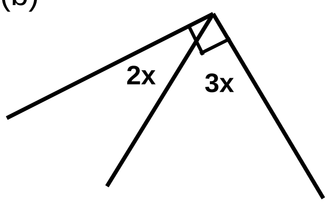
---

5. Calculate the value of ***x*** in each of the following:

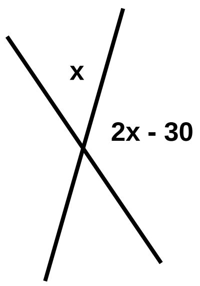
(a).



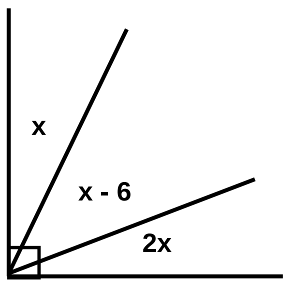
(b)



(c)



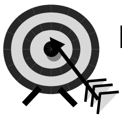
(d)



CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3

Lesson 4: Intersecting Lines

You learnt about complementary and supplementary angles in the previous lesson. You also learnt to work out the complement and supplement of given angles.



In this lesson, you will:

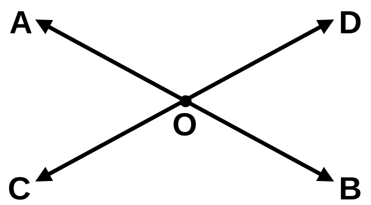
- define intersecting lines
- identify properties to find the size of missing angles formed by intersecting lines

Let us start with the definition of intersecting lines.

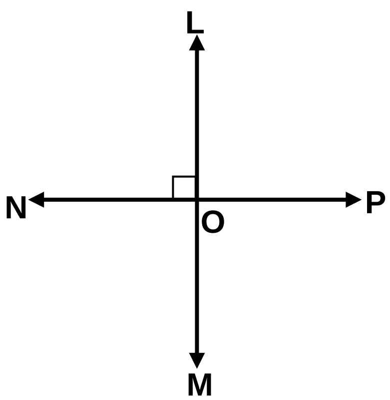
Intersecting lines are two lines that cut or cross over each other at a particular common point. The common point where the two lines meet is called the **point of intersection**.

The word '**intersect**' means '**to cut across'**.

In the figure below are two lines **AB** and **CD** which intersect each other at point **O.**



Now look at this figure.

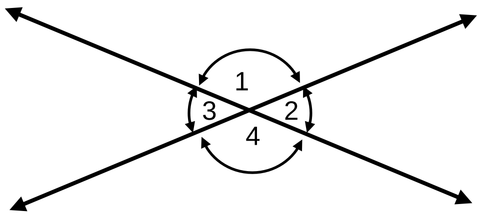


Lines **LM** and **NP** intersect at point **O** forming a right angle. We say line **LM** is perpendicular to line **NP**. We write it as **LM** $\perp$ **NP**.

Two intersecting lines are **perpendicular** if they form a right angle. The symbol '$\perp$' denotes 'is perpendicular to'.

Angles formed by Intersecting Lines

When two lines intersect, they usually form four angles at the point of intersection.



In the figure above, the four angles formed by the two intersecting lines are:

$\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$

The following angle pairs are formed when two straight lines meet:

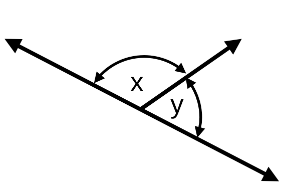
1. Adjacent angles
2. Vertical angles

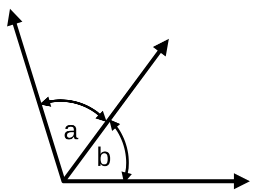
Adjacent angles are two angles that have a common side. They are the pair of angles that are next to each other.

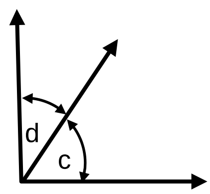
In the figure above, there are four adjacent angles. These are:

(a) $\angle 1$ and $\angle 2$
(b) $\angle 2$ and $\angle 4$
(c) $\angle 1$ and $\angle 3$
(d) $\angle 3$ and $\angle 4$

Here are other examples of adjacent angles.







Look at these diagrams.

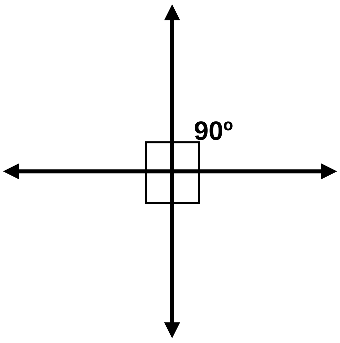


Diagram A

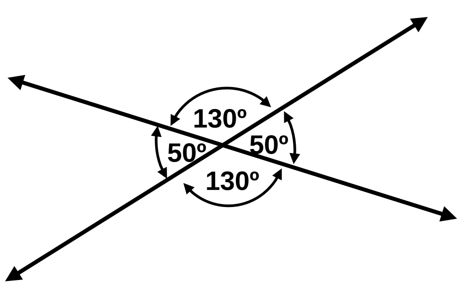


Diagram B

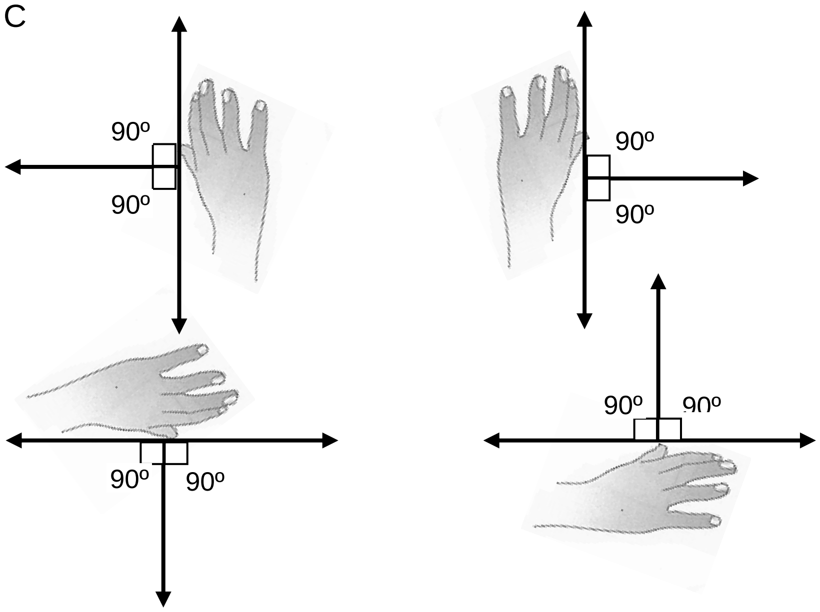
What do you notice about the measures of the adjacent angles?

In Diagram A, the measures of the adjacent angles are 90^o and 90^o.
In Diagram B, the measures of the adjacent angles are 130^o and 50^o.

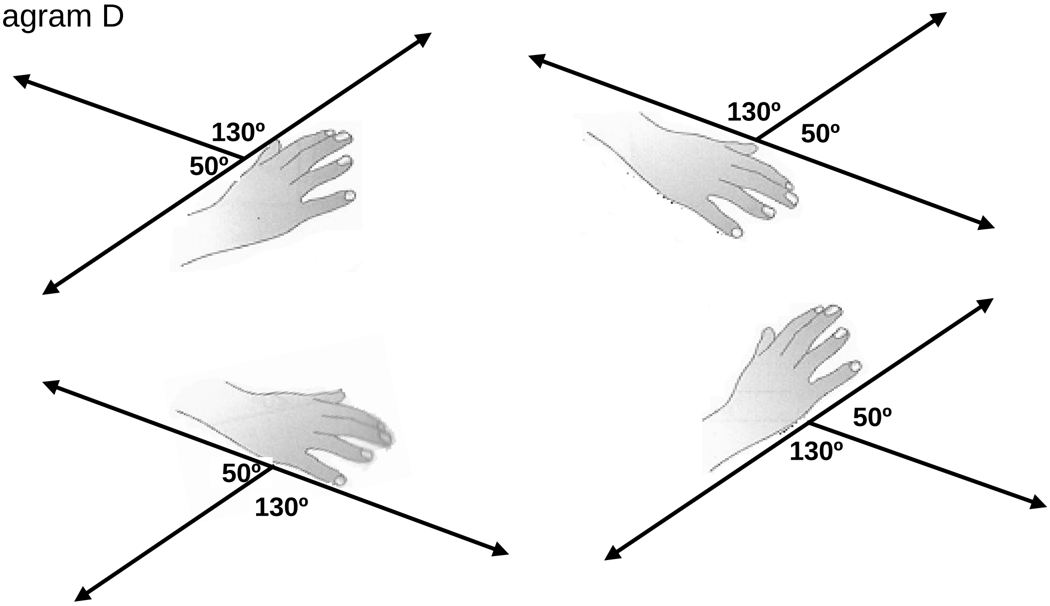
The **sum** of all the pairs of adjacent angles formed by two intersecting straight line is **180º**.

Now, by progressively covering up each of the arms of the angles in both Diagram C and D, we get the series of figures below. Each of these figures shows a pair of adjacent supplementary angles..

For Diagram C



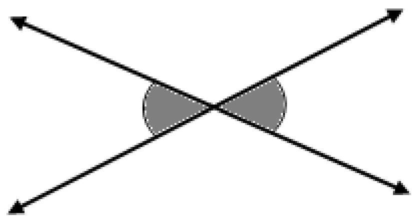
For Diagram D



We call each pair of angles a **linear pair** because the two non-common sides form a straight line.

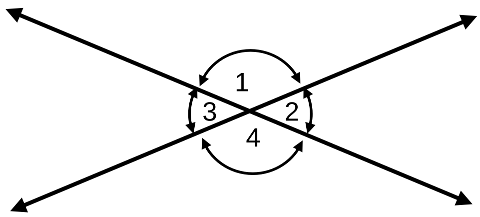
A **linear pair of angles** is a pair of angles that are adjacent and supplementary.

Now look at this diagram.



The shaded angles are called **vertical angles**. The word 'vertical' in this case means they '**share the same vertex** (corner point), not the usual meaning up-down.

Vertical angles are the angles that are opposite each other when two straight lines intersect or cross over each other.



In the figure above, there are two pairs of vertical angles: these are:

(a) $\angle 1$ and $\angle 4$

(b) $\angle 2$ and $\angle 3$

Now, look at these diagrams:

Diagram C

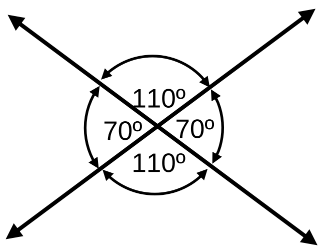
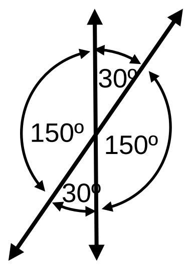


Diagram D



What do you notice about vertical angles?

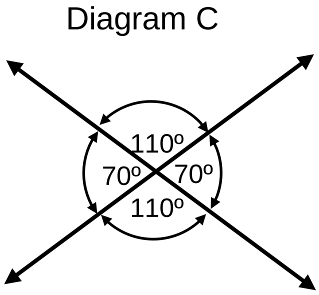
Diagram C: Vertical angles are 70^{o} and 70^{o}; 110^{o} and 110^{o}.

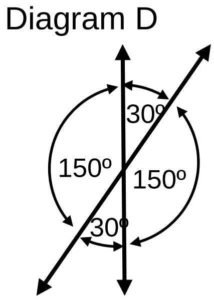
Diagram D: Vertical angles are 150^{o} and 150^{o}; 30^{o} and 30^{o}.

In both diagrams C and D, Vertical angles are **EQUAL**. This is **true** for any pair of Vertical angles.

Vertical angles are equal.

Again, look at the four angles formed around the point of intersection.





Now, add up all the angles formed around the point of the intersections.

Diagram C: 110º + 70º + 110º + 70º = 360º

Diagram D: 150 + 30 + 150 + 30 = 360º

What do you notice about the sum of the angles in both diagrams C and D?

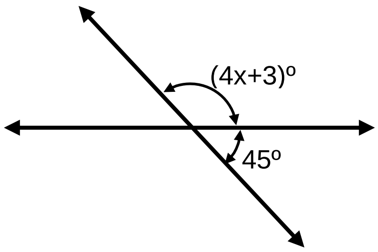
You notice that the **sum** of the angles in both diagrams is **360º**. This is true for all **angles at a point.**

The **sum of the angles** around the point of intersection is 360º.

Now using the knowledge learnt, we can calculate and solve for the measure of an unknown angle.

Example 1

Find the value of x at the right.



Solution:

We learned earlier that two angles that share a leg are adjacent, and if they are formed by straight lines they will be supplementary.

So we know that the 45º angle and the angle with the x are linear pairs because they are adjacent and supplementary.

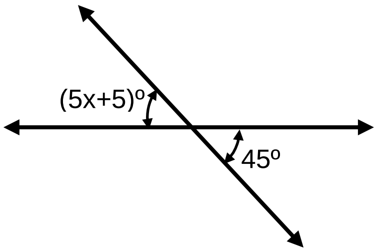
Using this information, we can form an equation and solve:

$$(4x + 3)^\circ + 45^\circ = 180^\circ$$
$$(4x + 3)^\circ = 180^\circ - 45^\circ$$
$$(4x)^\circ + 3^\circ = 135^\circ$$
$$(4x)^\circ = 135^\circ - 3^\circ$$
$$(4x)^\circ = 132^\circ$$
$$x = \frac{132}{4} = 33^\circ$$

Therefore, the value of x is 33º.

Example 2

Find the value of x in the figure at the right.



Solution:

We learned earlier that two angles that are formed by intersecting lines and are opposite to each other are vertical angles.

So we know that the 45º angle and the angle with the x are equal because vertical angles are always equal.

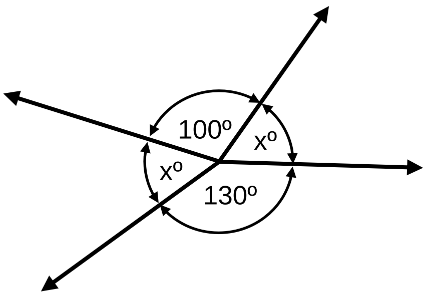
Using this information we can set up an equation and solve:

$$(5x + 5)^{\circ} = 45^{\circ}$$
$$(5x)^{\circ} = 45^{\circ} - 5^{\circ}$$
$$(5x)^{\circ} = 40^{\circ}$$
$$x = \ = \frac{40^{\circ}}{5} = 8^{\circ}$$

Therefore, the value of x is 8º.

Example 3

Calculate the value of x in the figure at the right.



Solution:

We learned earlier that intersecting lines form angles at the point of intersection and that the sum of the angles at the point is 360º.

Using this information we can set up an equation and solve:

$$x^{\circ} + x^{\circ} + 100^{\circ} + 130^{\circ} = 360^{\circ}$$
$$2x^{\circ} + 230^{\circ} = 360^{\circ}$$
$$2x^{\circ} = 360^{\circ} - 230^{\circ}$$
$$2x^{\circ} = 130^{\circ}$$
$$x = \frac{130^{\circ}}{2} = 65^{\circ}$$

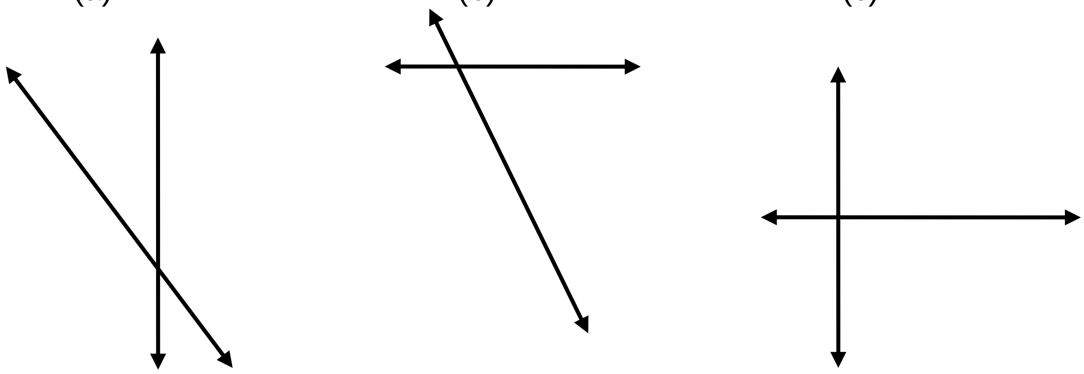
Therefore, the value of x is 65º.

NOW DO PRACTICE EXERCISE 4

Practice Exercise 4

1. Which of the following pairs of lines is perpendicular?

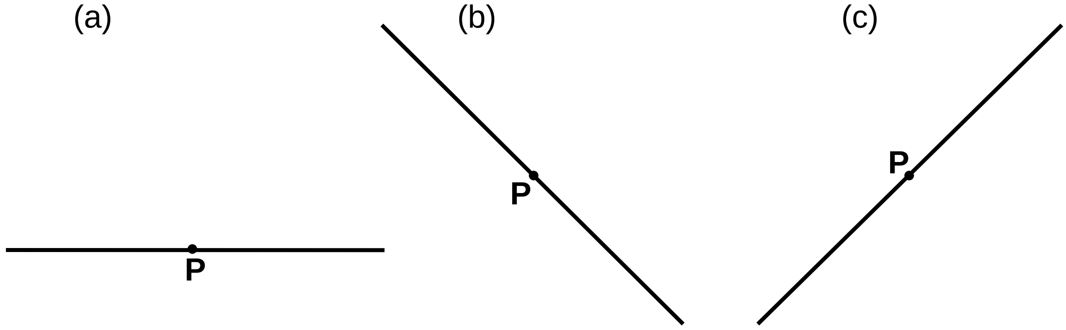
(a) (b) (c)



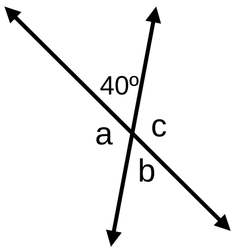
Answer: ____________

2. Use your ruler and a protractor to draw a line perpendicular to each line at point **P** below.

(a) (b) (c)



3. Study this diagram to help you answer the questions below.



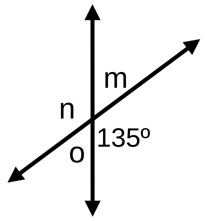
Fill in the blank spaces.

(a) Vertical angles are ____________ so the value of **b** is ___________.

(b) Angles **b** and **c** are adjacent angles, so **b + c** = ___________

(c) **b + c** = 180º **b** = 40º, so **c** = ____________.

4. Use the diagram to fill the blank spaces.



(a) The value of **n** = ________

(b) **m**+ 135º= 180º

_____ + 135º = 180º.

(c) **m** is vertically opposite **o,** so the value of **o** = ________.

(d) All angles at the intersecting point add up to ________

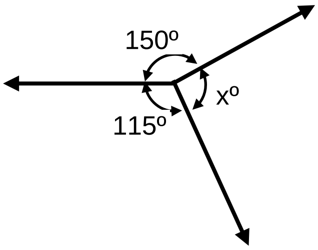
so **m** + **n** + **o** + 135º = _________

________ + ________ + ________ + 135º = ________.

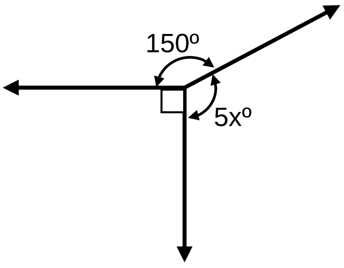
---

5. Find the value of x in the following figures.

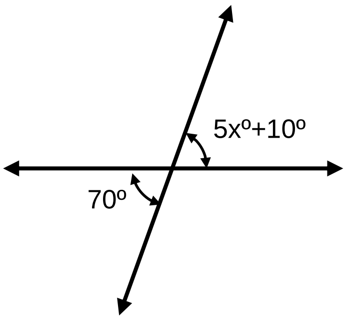
(a)



(b)



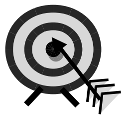
(c)



CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 5: Parallel Lines

You learnt about the meaning of intersecting lines in the last lesson. You also learnt properties of the angles formed by intersecting lines.



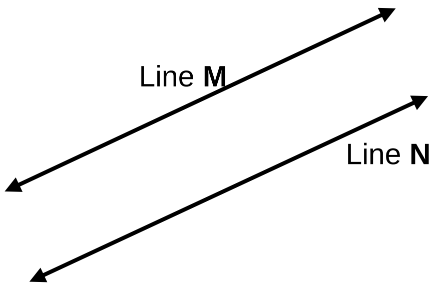
In this lesson, you will:

- define parallel lines
- identify properties of parallel lines and the notation used to denote parallel lines
- draw parallel lines.

You were introduced to parallel lines in your Grade 8 Mathematics. In this lesson, you will extend further your knowledge of parallel lines.

First, let us define parallel lines.

Parallel lines are lines that do not meet or cross each other however far they are extended.

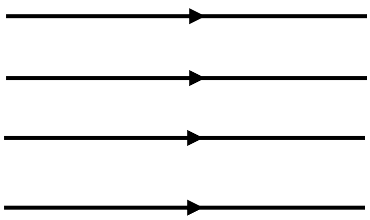


Lines **M** and **N** are parallel. We say Line **M** is parallel to Line **N**. We write this as **M//N.**

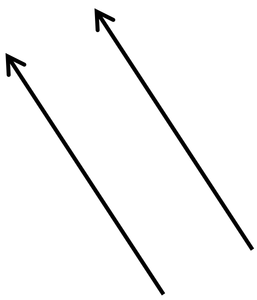
The symbol ***//*** is used to denote parallel lines. It means "is Parallel to".

Here are some more examples of parallel lines.

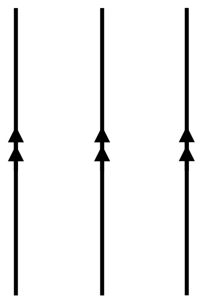
(a)



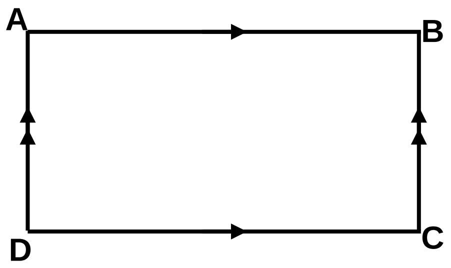
(b)



(c)

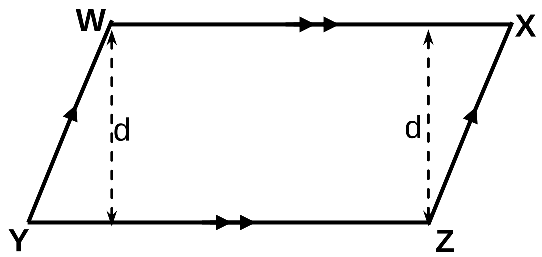


We use single arrowheads or double arrowheads to show that lines are parallel. If there are two different sets of parallel lines in a diagram, then we use single arrowheads on one set of parallel lines and double arrowheads on the other set as shown here.



Arrow heads are placed on lines that are parallel.

Now, look at this diagram. This will help you identify the properties of parallel lines.



Here are the properties of parallel lines.

1. Parallel lines do not meet or never intersect each other. You can see that;

$\overline{\mathbf{WX}}$ does not meet $\overline{\mathbf{YZ}}$
$\overline{\mathbf{WY}}$ does not meet $\overline{\mathbf{XZ}}$

2. Parallel lines go in the same direction. You can see

$\overline{\mathbf{WX}}$ and $\overline{\mathbf{YZ}}$ go in the same direction
$\overline{\mathbf{WY}}$ and $\overline{\mathbf{XZ}}$ go in the same direction

3. Parallel lines are the same distance apart between them from the start to the end. This means that the distance between the parallel lines will be the same at all points.

$\overline{\mathbf{WX}}$ and $\overline{\mathbf{YZ}}$ are the same distance apart between them from point **W** to point **Z** and at any point along **W** and **Z**.

4. Parallel lines are marked by the same symbol. The symbols indicate that they are parallel.

$\overline{\mathbf{WX}}$ is parallel to $\overline{\mathbf{YZ}}$.
$\overline{\mathbf{WY}}$ is parallel to $\overline{\mathbf{XZ}}$.

We use this symbol ***//*** to indicate parallel lines.

$\overline{\mathbf{WX}} /\!/ \overline{\mathbf{YZ}}$.
$\overline{\mathbf{WY}} /\!/ \overline{\mathbf{XZ}}$.

Now let us draw parallel lines.

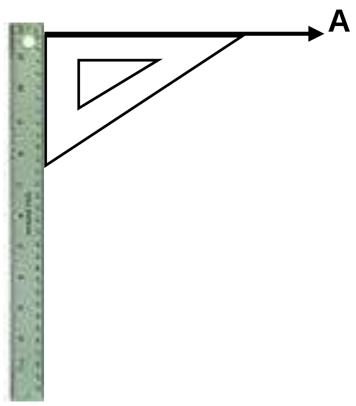
You will need a set square, a ruler and a pencil in this activity.

Drawing Parallel lines

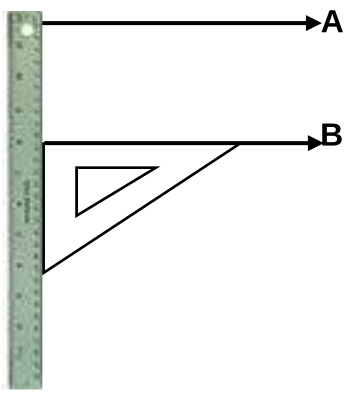
We use the method shown below when using set-squares to draw parallel lines.

1. First, draw a horizontal line and label it **A.**

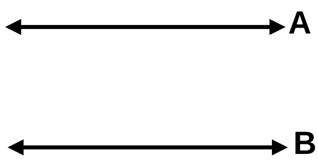
2. Then place the horizontal edge of the set-square at one end of line **A** and place a ruler so that it touches the vertical edge of the set-square by holding the ruler firmly.



3. Slide the set-square down the ruler and draw a line along the horizontal edge of the set-square. Label the second line **B**.



4. Remove the ruler and the set-square. You should have a pair of parallel lines.

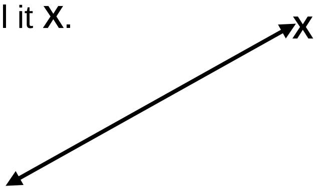


We say line **A** is parallel to line **B** or **A // B**.

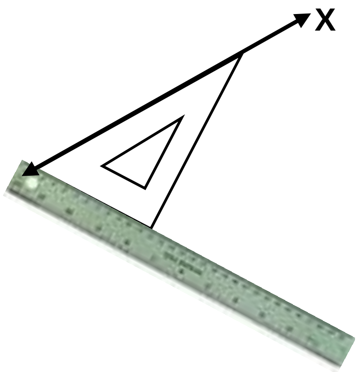
We can also draw slanting parallel lines.

We use the same method we used to draw horizontal parallel lines.

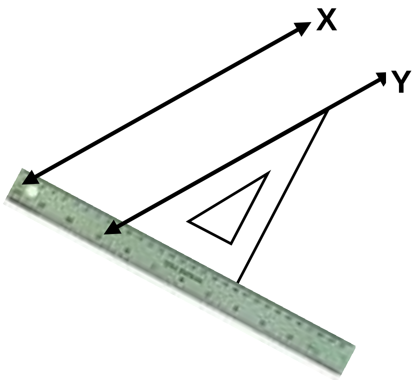
1. Draw a straight line and label it **X**.



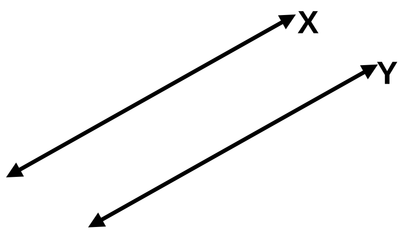
2. Put the set-square against the line and a ruler against the set-square.



3. Slide the set-square along the ruler until it is some distance away from line **X.** Draw a line along the edge of the set-square and label it **Y**.



4. When you take your ruler and set-square away, you will have a pair of parallel lines.



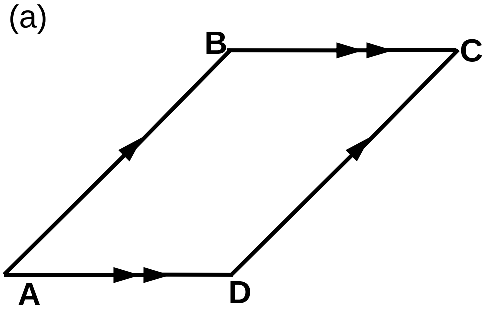
We say that line **X** is parallel to line **Y**. That is **X // Y**.

NOW DO PRACTICE EXERCISE 5

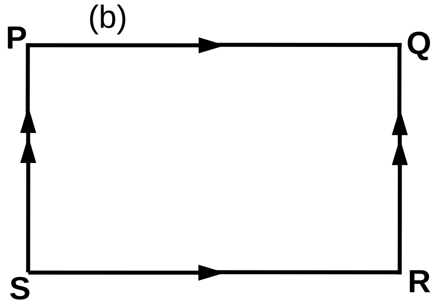
Practice Exercise 5

1. Write down the pairs of parallel lines in the following shapes.
(Use the symbol **//** 'parallel to')

(a)



(b)



(a) (i) ____________________

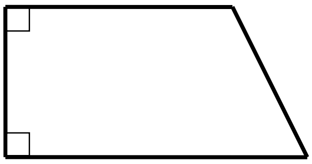
(ii) ____________________

(b) (i) ____________________

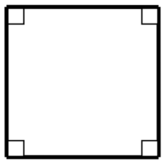
(ii) ____________________

2. Show parallel sides in the following shapes by drawing arrows.

(a)

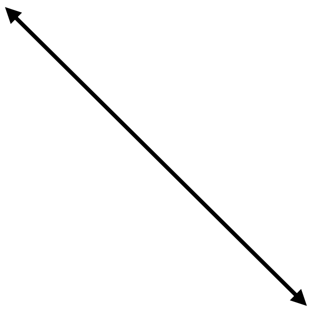


(b)



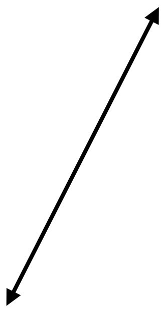
3. Draw a line parallel to each of the following lines.

(a)



(b)

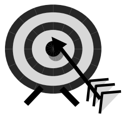
(c)



CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 6: Reasoning With Parallel Lines

You learnt the meaning of parallel lines and their properties in the last lesson. You also learnt to draw parallel lines using a ruler and a set-square.



In this lesson, you will:

- define a transversal line
- identify the angles formed by the transversal and a pair of parallel lines
- identify the relationship between the angles formed
- calculate the missing angles formed by the pair of parallel line and the transversal.

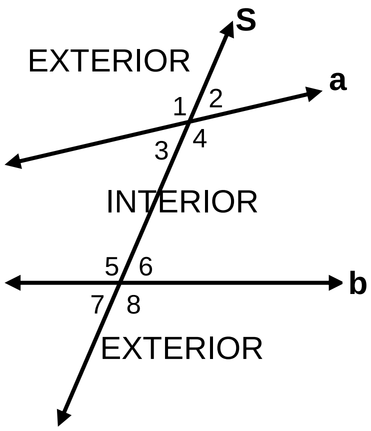
First, let us define a transversal.

A **transversal** is a line that intersects two or more lines (in the same plane).

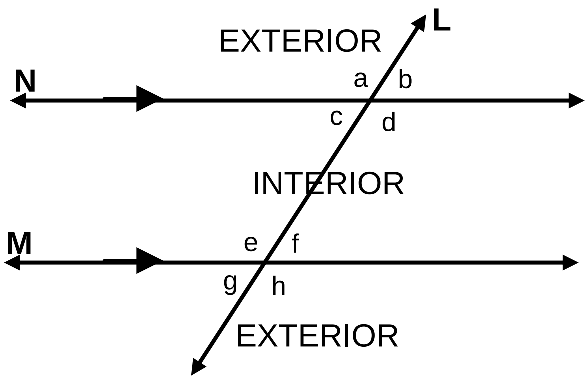
When lines intersect, angles are formed in several locations. Certain angles are given "**names**" that describe "**where**" the angles are located in relation to the lines. These names describe angles whether the lines involved are parallel or not parallel.

Look at the following diagrams.

When the lines are not parallel



When the lines are parallel…



$\overleftrightarrow{N} \parallel \overleftrightarrow{M}$

Remember that:

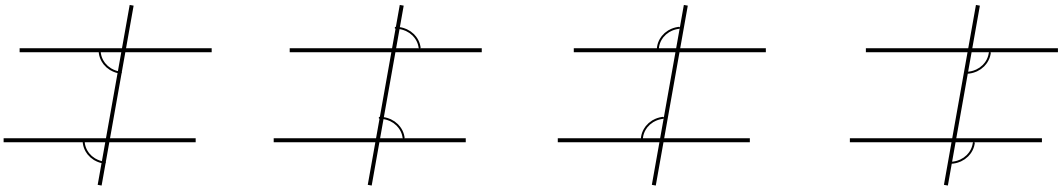
- the word **INTERIOR** means **BETWEEN** the lines
- the word **EXTERIOR** means **OUTSIDE** the lines
- the word **ALTERNATE** means **"alternating sides"** of the transversal.

Angles and Parallel Lines

Parallel lines create situations in which some angles formed by intersecting lines are equal. We will outline which angles are equal in this lesson.

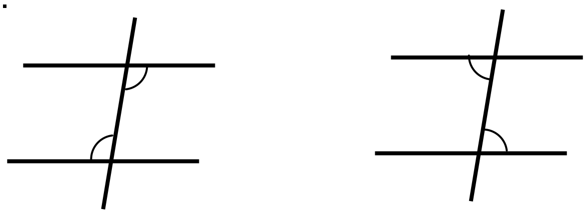
The special relationships that occur when two lines are parallel are set out in the following rules.

- When a pair of parallel lines is cut by a transversal, four pairs of **corresponding angles** are formed.



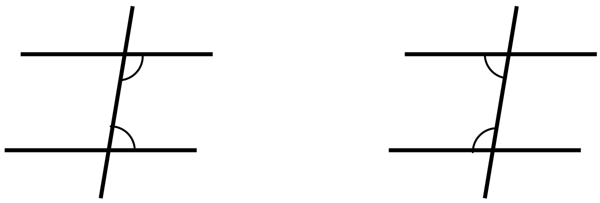
Corresponding angles are pairs of angles, one exterior and one interior angle on the same side of the transversal and not adjacent.

- When a pair of parallel lines is cut by a transversal, two pairs of equal **alternate angles** are formed.



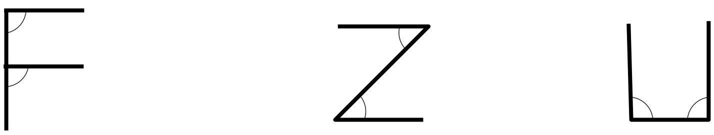
Alternate angles are pairs of interior angles on the opposite side of the transversal and not adjacent.

- When a pair of parallel lines is cut by a transversal, two pairs of **co-interior angles** are formed. Co-interior angles are supplementary.



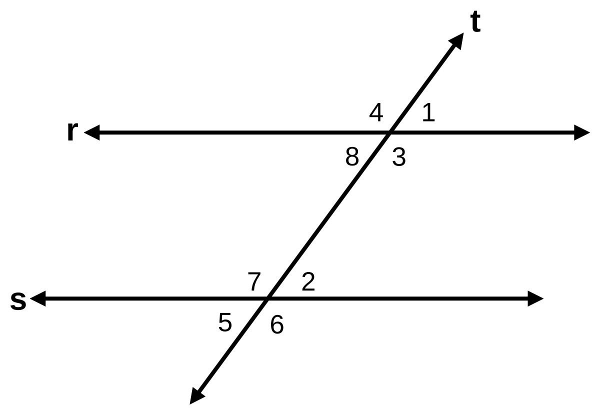
Co-interior angles are two interior angles at the same side of a transversal.

You should learn to recognize pairs of corresponding, alternate and co-interior angles. Drawing the 3 blocks of letters F, Z and U will help you to remember these angles.



corresponding alternate Co-interior

Let us consider the following information on the figure to better understand all these rules.



In the figure, lines **r** and **s** are parallel, we write **r // s,** and line **t** intersecting both lines **r** and **s** is called the t**ransversal.**

We identify angles by their positions in this diagram.

For example:

$\angle 1$ is an exterior angle and $\angle 2$ is an interior angle both on the same side of the transversal. Such angles are called **corresponding angles**. Similarly, we have $\angle 3$ and $\angle 6$, $\angle 4$ and $\angle 7$, and $\angle 8$ and $\angle 5$ as **corresponding angles**.

$\angle 8$ and $\angle 2$, $\angle 3$ and $\angle 7$ are on opposite sides of the transversal and between (interior) the parallel lines and not adjacent. We call these angles **alternate interior angles.**

$\angle 1$ and $\angle 5$, $\angle 4$ and $\angle 6$ are on opposite sides of the transversal and above and below (exterior to) the parallel lines and not adjacent. We call these angles **alternate exterior angles.**

$\angle 3$ and $\angle 2$, $\angle 8$ and $\angle 7$ are two pairs of interior angles on the same side of the transversal. We call these angles **co-interior angles**.

A direct result of the famous **Parallel Postulate** is that corresponding angles are equal. Accepting this fact gives us the following relationships:

$\angle 1 = \angle 2$, $\angle 3 = \angle 6$, $\angle 4 = \angle 7$, and $\angle 8 = \angle 5$

Using these facts, we can show that other angles must also be equal.

Since $\angle 2$ is supplementary to $\angle 6$ and $\angle 1$ is supplementary to $\angle 4$, then $\angle 6 = \angle 4$ because they are supplementary to equal angles. $\angle 4$ and $\angle 6$ are **alternate exterior angles.**

Since $\angle 1$ is supplementary to $\angle 3$ and $\angle 7$ is supplementary to $\angle 2$, then $\angle 7 = \angle 3$ because they are supplementary to equal angles. $\angle 7$ and $\angle 3$ are **alternate interior angles.**

Summarising these we have the following:

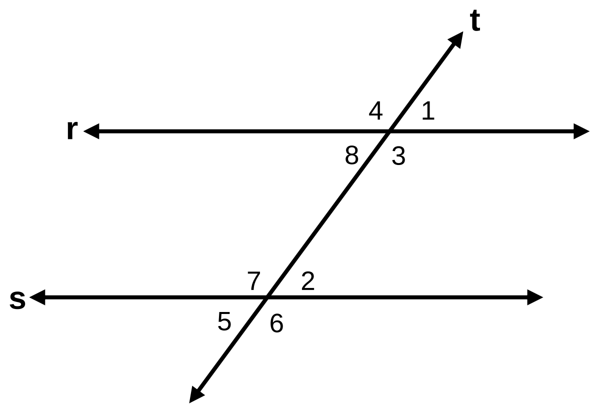
- Corresponding angles formed by parallel lines and a transversal are equal.
- Alternate interior angles formed by parallel lines and a transversal are equal Alternate exterior angles formed by parallel lines and a transversal are equal
- Co-interior angles are supplementary.

Using the knowledge on angles formed by parallel lines cut by a transversal, we will try to solve for the missing angle.

Now look at the examples.

Example 1

Using the diagram below, suppose that $\angle 1$ has a measure of 55º. What are the measures of the other angles in the diagram? You may assume that Lines **r** and **s** are parallel.



Solution:

$\angle 8 = 55º$, since it is vertical to $\angle 1$.

$\angle 2 = 55º$, since it is a corresponding angle to $\angle 1$.

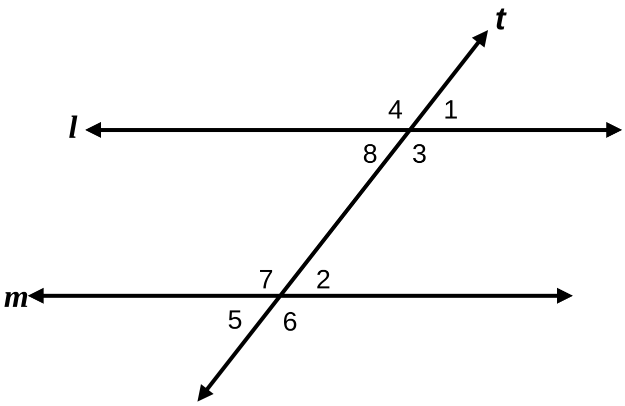
$\angle 5 = 55º$, since it is vertical with $\angle 2$, and also since it is an alternate exterior angle to $\angle 1$.

The other angles are all supplementary to angles 1, 2, 5 and 8. Therefore they have a measure of 125º.

We can also conclude that $\angle 1 = \angle 5$, since they are alternate exterior angles.

Example 2

In the diagram below, assume that Lines ***l*** and ***m*** are parallel. Line ***t*** is a transversal. If $\angle 7$ has a measure of 120º, what are the measures of the other angles?



Solution:

$\angle 6$ = 120º since it is vertical to $\angle 7$

$\angle 3$ = 120º since it is a corresponding angle with $\angle 7$

$\angle 4$ = 120º since it is vertical with $\angle 3$

All the other angles have a measure of 60º. For example, $\angle 1$ is supplementary to $\angle 4$ and must add to 180º with $\angle 4$.

This makes all the corresponding, vertical, and alternate exterior/interior angles 60º.

NOW DO PRACTICE EXERCISE 6

Practice Exercise 6

1. Using the diagram below, identify the given angle pairs as **alternate interior, alternate exterior, consecutive interior, consecutive exterior, corresponding, or none of these.**

(a) $\angle 1$ and $\angle 7$

(b) $\angle 2$ and $\angle 8$

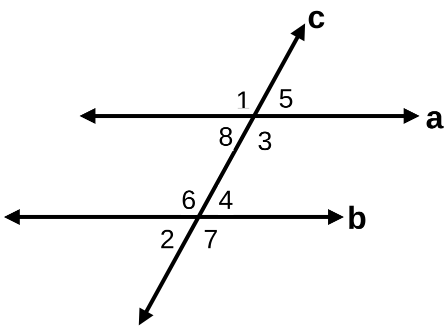
(c) $\angle 3$ and $\angle 4$

(d) $\angle 4$ and $\angle 8$

(e) $\angle 3$ and $\angle 8$

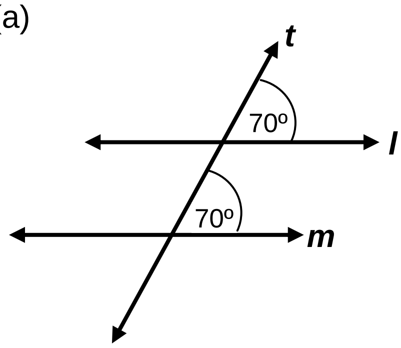
(f) $\angle 3$ and $\angle 2$

(g) $\angle 5$ and $\angle 7$

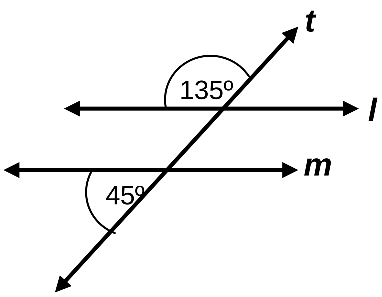


2. Name the following pairs of angles.

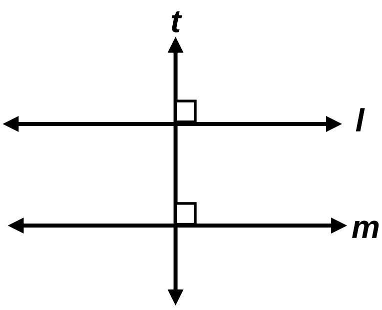
(a)



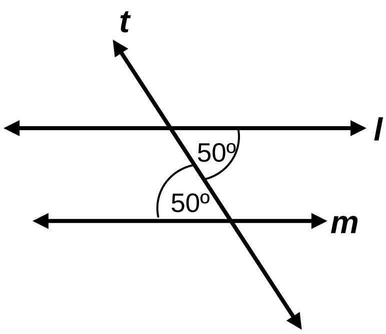
(b)



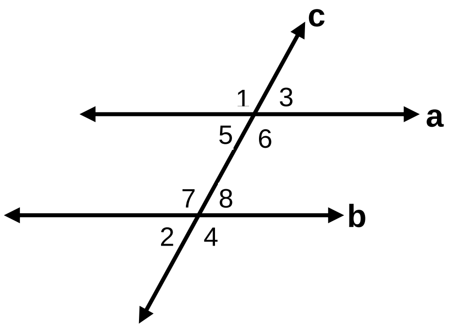
(c)



(d)

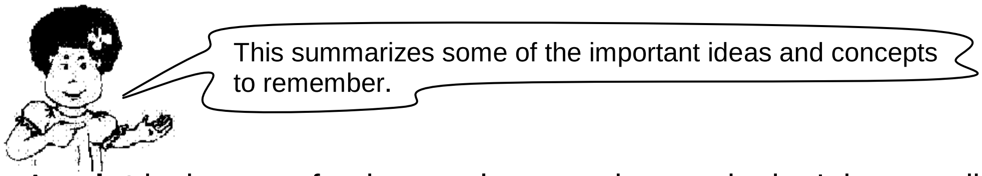


3. In the figure below, *a* // *b* and $m\angle 1 = 117°$. Find the measure of each of the other numbered angles in the diagram.



CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4

TOPIC 1: SUMMARY



- A **point** is the most fundamental geometric magnitude. It has **no** dimension. It is a fixed location in space with no definite form size or shape. It is represented by a **dot** like a period.
- **Collinear points** is a set of points that lie in a straight line
- **Non-collinear points** are set of points that do not lie on the same line**.**
- **Coplanar points** are set of points that lie on the same plane.
- **Non-coplanar points are** set of points that do not lie on the same plane.
- A **plane** is a flat surface that extends infinitely in all directions. It has two dimensions; its length and its width.
- A **line** is geometric magnitude formed by a set of points moving indefinitely in any directions along a straightedge. It has **one** dimension, its **length**.
- A **line segment** is a part of a **line** that is bounded by two distinct end points, and contains every point on the **line** between its end points.
- A **ray** is also a piece of a line, except that it has only one endpoint and continues forever in one direction. It could be thought of as a half-line with an endpoint.
- An **angle** is a plane figure formed by two rays or lines that diverge or extend from a common endpoint and do not lie on a straight line. The common endpoint is the **vertex** and the two rays are the **sides**.
- **Complementary angles** are any two angles whose sum is 90^o and make a right angle.
- If two angles add up to 90^o, then they '**complement'** each other.
- **Supplementary angles** are any two angles whose sum is 180^o and make a straight angle.
- If two angles add up to 180^o, then they **'supplement'** each other**.**
- **Intersecting lines** are two lines that cut or cross over each other at a particular common point. The common point where the two lines meet is called the **point of intersection**.
- **Adjacent angles** are two angles that have a common side. They are the pair of angles that are next to each other.
- **Vertical angles** are the angles that are opposite each other when two straight lines intersect or cross over each other.
- **Parallel lines** are lines that do not meet or cross each other however far they are extended.
- A **transversal** is a line that intersects two or more lines (in the same plane).
- **Corresponding angles** are pairs of angles, one exterior and one interior angle on the same side of the transversal and not adjacent.
- **Alternate angles** are pairs of interior angles on the opposite side of the transversal and not adjacent.
- **Co-interior angles** are two interior angles at the same side of a transversal.

REVISE LESSONS 1- 6. THEN DO TOPIC TEST 1 IN ASSIGNMENT BOOK 5.

ANSWERS TO PRACTICE EXERCISES 1- 6

Practice Exercises 1

1. (a) NO
   (b) Point **S**
   (c) Lines **AB** and **XY**
   (d) Point **Q**
   (e) $\angle$**1** or $\angle$**ASK**
   (f) Points **A** and **S**

2. (a) vertex of an angle
   (b) Collinear points
   (c) midpoint of a line segment

3. vertex

4. (a) $\angle$**ADB**
   (b) $\angle$**DCB**

5. Six (6) segments

6. $\overline{DE}$, $\overline{DF}$, $\overline{DG}$, $\overline{EF}$, $\overline{EG}$, $\overline{FG}$

Practice Exercises 2

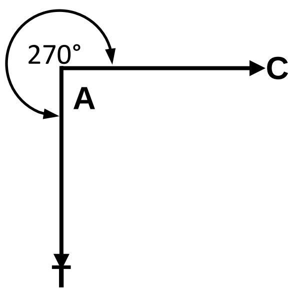
1. (a) obtuse $\angle$
   (b) acute $\angle$
   (c) right $\angle$
   (d) acute $\angle$
   (e) reflex $\angle$
   (f) obtuse $\angle$

2. (a) acute $\angle$
   (b) obtuse $\angle$
   (c) 1 rotation or 1 full circle
   (d) straight $\angle$
   (e) reflex $\angle$
   (f) right $\angle$
   (g) reflex $\angle$
   (h) acute $\angle$

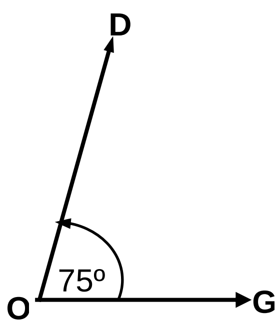
3. $\angle$**Y** , $\angle$**2** , $\angle$**XYZ** or $\angle$**ZYX**

4. (a) 70º (b) 42º (c) 105º
   (d) 140º (e) 30º

5. (a)



(b)



Practice Exercises 3

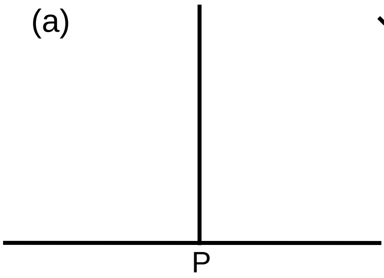
A. 1. (a) 129°
2. (a) 75°
3. (b) 37°
4. (a) Complementary angles
5. (d) 55°

B. 1. 65°
2. 48°
3. 45° and 135°
4. 126°
5. (a) 15° (b) 18° (c) 70° (d) 24°

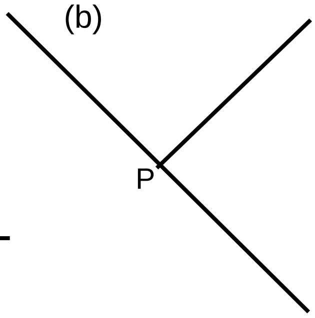
Practice Exercises 4

1. (c)

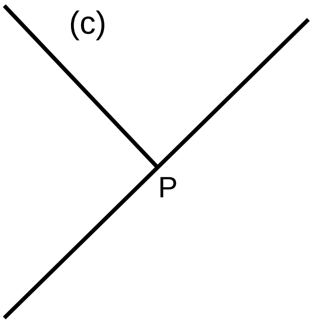
2. (a)



(b)



(c)



3. (a) Vertical angles are ___**equal**___ so the value of **b** is ___40°___.

(b) Angles **b** and **c** are adjacent angles, so **b + c** = ___180°___

(c) **b + c** = 180° **b** = 40°, so **c** = ___140°___.

4. (a) The value of **n** = ___135°___

(b) **m**+ 135°= 180°

___45°___ + 135° = 180°.

(c) **m** is vertically opposite **o,** so the value of **o** = ___45°___.

(d) All angles at the intersecting point add up to ___360°___

so **m** + **n** + **o** + 135° = ___360°___

___45°___ + ___135°___ + ___45°___ + 135° = ___360°___.

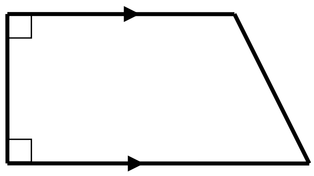
5. (a) 95° (b) 24° (c) 12°

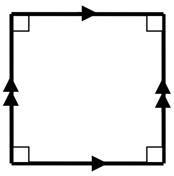
Practice Exercises 5

1. (a) (i) BC // AD (ii) AB // DC

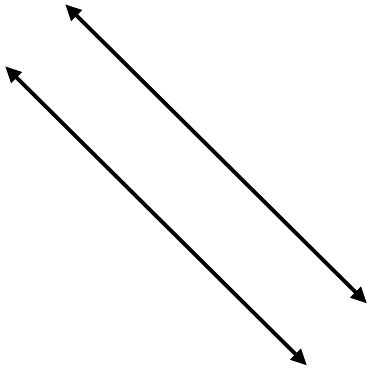
   (b) (i) PQ // SR (ii) PS // QR

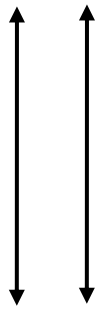
2. (a) (b)

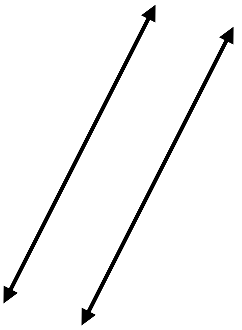




3. (a) (b) (c)







Practice Exercises 6

1. (a) alternate exterior $\angle$s
   (b) corresponding $\angle$s
   (c) consecutive interior $\angle$s
   (d) alternate interior $\angle$s
   (e) none of these
   (f) none of these
   (g) consecutive exterior $\angle$s

2. (a) corresponding $\angle$s
   (b) consecutive exterior $\angle$s
   (c) corresponding $\angle$s
   (d) alternate interior $\angle$s

3. $\angle 2 = 63^o$ $\angle 5 = 63^o$ $\angle 8 = 63^o$
   $\angle 3 = 63^o$ $\angle 6 = 117^o$
   $\angle 4 = 117^o$ $\angle 7 = 117^o$

END OF TOPIC 1

TOPIC 2

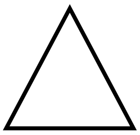
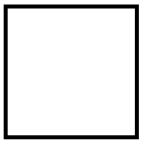
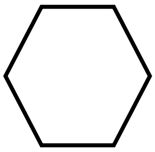
POLYGONS

TOPIC 2: POLYGONS

Introduction

One of the most important types of geometric figures is a polygon.

Look at these plane shapes.

These shapes have a general name. They are called **polygons**.

Polygons are all around. Take a look around you in your house, in your school, in your work place, or in an amusement park. You will find many real life examples of polygons.

Below are some of them.

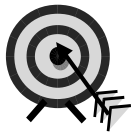




In this topic, you will describe polygons. You will identify and name different types of polygons and their properties especially triangles and quadrilaterals. You will also solve problems involving polygons.

Lesson 7: Types of Polygons

In the previous topic, you learnt the basic ideas in geometry and also the different angles and angle relationships.



In this lesson you will:

- define polygon, regular and irregular polygons
- name and classify polygons according to the number of sides
- identify and draw concave and convex polygons
- find the angle sum of polygons
- find the size of an interior angle of a polygon

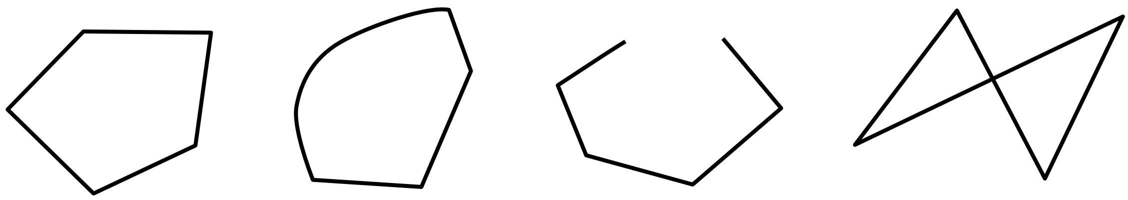
Let us start with the meaning of a polygon.

> A **polygon** is a closed figure in a plane made by joining three or more straight line segments,

Note: Strictly speaking, a polygon does not include its interior (the space inside the polygon).

The word ‘**polygon**’ came from the two Greek words ‘**poly**’ meaning ‘**many**’ and ‘**gon**’ meaning ‘**angle**”. Polygon means ‘many angles’. Along its angles, a polygon also has sides and vertices. ‘Tri’ means ‘three’, so the simplest polygon is called the triangle, because it has three angles. It also has three sides and three vertices. A triangle is always coplanar, which is not true to many of the other polygons.

Below are examples of figures that are polygons and not polygons.

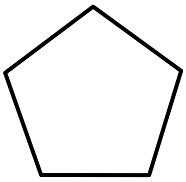


a polygon | **Not a polygon (has a curve)** | **Not a polygon (Open not closed)** | **Not a polygon (line segments intersects)**

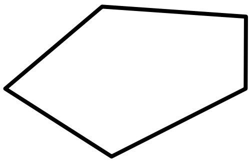
There are two important properties of polygons. Every polygon is either regular or irregular and either concave or convex.

Regular and Irregular

> If all angles are equal and all sides are equal, then it is a **regular polygon**, otherwise it is an **irregular polygon.**



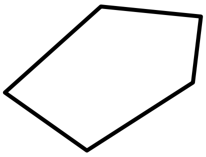
Regular



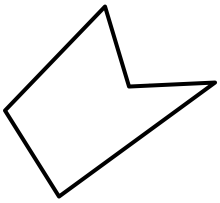
Irregular

Concave or Convex

A **convex** polygon has no angles pointing inwards. More precisely, no internal angle can be more than 180°. If any internal angle is greater than 180° then the polygon is **concave**. (*Think: concave has a "cave" in it*).



Convex
It has all angles less than 180º.



Concave
It has one angle bigger than 180º.

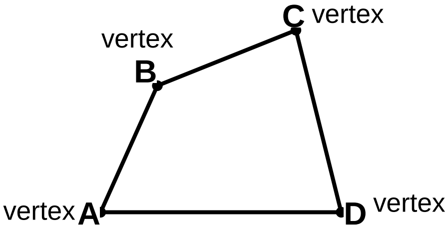
Polygons are usually classified according to how many sides they have. The table below shows the name and number of sides of polygons.

Name	Sides	Name	Sides
Triangle	3	Octagon	8
Quadrilateral	4	Nonagon	9
Pentagon	5	Decagon	10
Hexagon	6	Undecagon	11
Heptagon	7	Dodecagon	12

Generally, a polygon with many sides can be called **n-gon**.

Identifying the parts of a polygon

The endpoints of the sides of polygons are called **vertices.** The singular form of **vertices** is **vertex**.



Vertices A, B, C and D

When naming a polygon, its vertices are named in consecutive order either clockwise or counterclockwise.

For example, the polygon above could be named:

- polygon ABCD start with A move clockwise
- polygon BCDA start with B move clockwise
- polygon CDAB start with C move clockwise
- polygon DABC start with D move clockwise

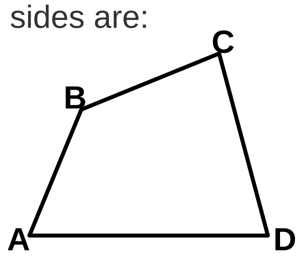
- polygon ADCB start with A move counter-clockwise
- polygon DCBA start with D move counter-clockwise
- polygon CBAD start with C move counter-clockwise
- polygon BADC start with B move counter-clockwise

It does not matter with which letter you begin as long as the vertices are named consecutively.

Consecutive sides are the **sides of a polygon** that share an end point.

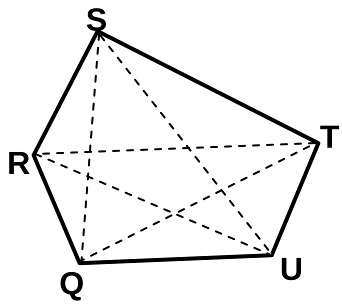
For example, for polygon ABCD, the consecutive sides are:

- $\overline{AB}$ and $\overline{BC}$
- $\overline{BC}$ and $\overline{CD}$
- $\overline{CD}$ and $\overline{DA}$
- $\overline{DA}$ and $\overline{AB}$



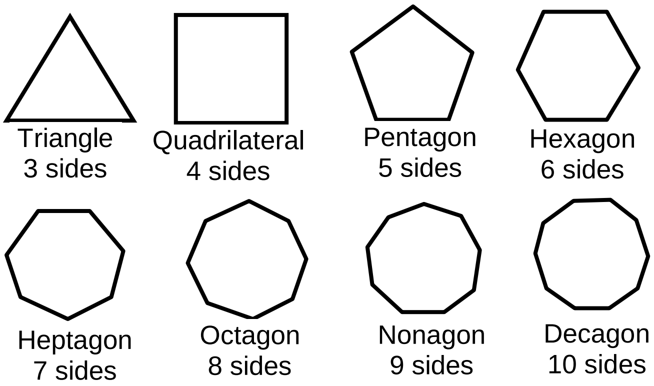
A **diagonal** of a polygon is any segment that joins two non-consecutive vertices.

The figure below shows a five-sided polygon **QRSTU**. $\overline{\mathbf{QS}}$, $\overline{\mathbf{SU}}$, $\overline{\mathbf{UR}}$, $\overline{\mathbf{RT}}$ and $\overline{\mathbf{QT}}$ are the diagonals in this polygon.



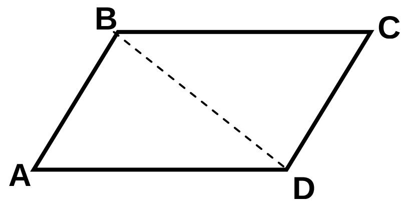
Earlier you learnt that if a polygon has all its angles equal and its sides equal, then the polygon is said to be regular. For a polygon to be regular, it must be both equiangular and equilateral at the same time it must also be convex. See the examples of regular polygons on the next page.

CONVEX REGULAR POLYGONS



Polygons can be divided into several non-overlapping triangles by drawing all the possible diagonals that can be drawn from one single vertex.

Let us try it with the quadrilateral shown below. From vertex B, we can draw only one diagonal to vertex D. A quadrilateral can therefore be divided into two triangles.



The interior angle sum of this quadrilateral can now be found by multiplying the number of triangles by 180º. Upon investigation, it is found that the number of triangles is always two less than the number of sides.

The **sum of the interior angles** of any polygon is **(n- 2)180º**.

The formula is:

$$\boxed{\mathbf{S = (n - 2)180^\circ}}$$

Where **S** = the sum of the interior angles
n = the number of sides of any polygon

To find the sum of the angles of the quadrilateral, we can use the formula. Substitute 4 (number of sides) for **n**, we then find that the sum of the interior angle **S** of a quadrilateral is 360º.

Working out: $\mathbf{S = (n - 2)180^\circ}$

$S = (4 - 2)180^\circ$

$S = (2)\ 180^\circ$

$S = 360^\circ$

The figure below illustrates this division using a heptagon (a seven-sided polygon. From vertex **M** we can draw 4 diagonals which divide the heptagon into 5 triangles. We multiply 5 times 180º to find the sum of the interior angles of a heptagon which is 900º.

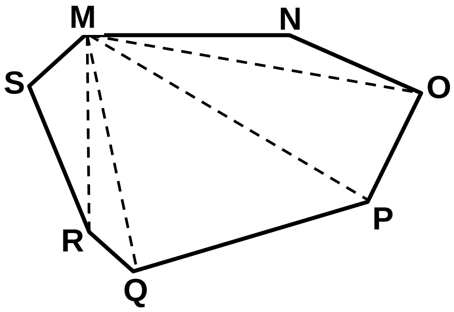
Working out:

$S = (n - 2)180^\circ$

$S = (7 - 2)180^\circ$

$S = (5)180^\circ$

$\mathbf{S = 900^\circ}$



Here is another example.

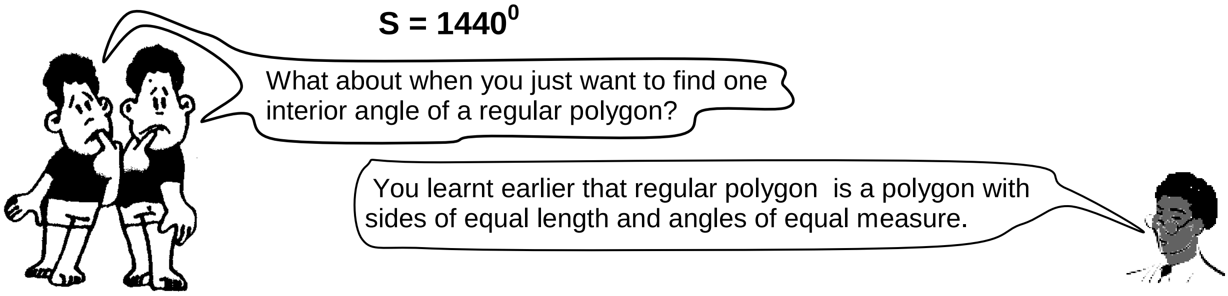
Find the sum of the interior angles of a regular decagon.

Solution: A decagon has 10 sides, so:

$S = (10 - 2)180^0$

$S = (8)180^0$

$\mathbf{S = 1440^0}$



In order to find the measure of a single interior angle of a regular polygon, we just divide the sum of the interior angles by the number of sides.

Each **interior angle** of any regular polygon of **n** sides is equal to the sum of the interior angles divided by the number of sides.

The formula is:

$$\textbf{Each Interior Angle} = \frac{(\mathbf{n-2})\mathbf{180^\circ}}{\mathbf{n}}$$

Example: To find the measure of an interior angle of a regular hexagon, which has 6 sides, apply the formula above as follows:

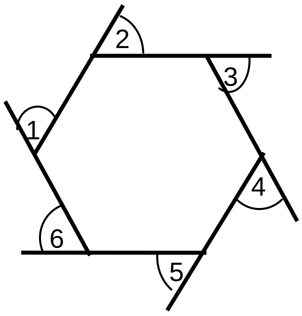
$$\text{Each Interior Angle} = \frac{(n-2)180^\circ}{n}$$

$$= \frac{(6-2)180^\circ}{6} = \frac{(4)180^\circ}{6} = \frac{720^\circ}{6} = 120^\circ$$

Therefore, each interior angle of a regular hexagon is 120º.

The Exterior Angle of a Polygon

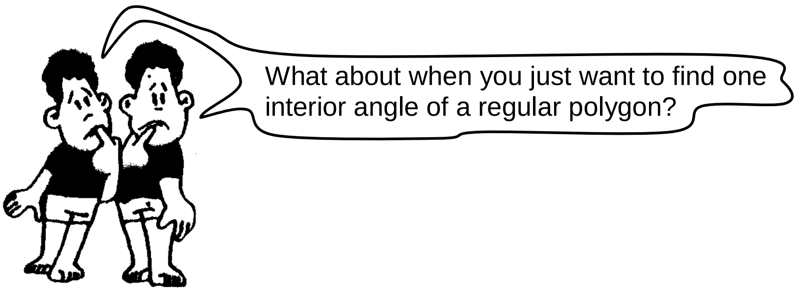
An **exterior angle** of a polygon is an angle outside the polygon formed by one of its sides and the extension of an adjacent side.



Finding the sum of the exterior angles of a polygon is simple. No matter what type of polygon you have the sum of the exterior angles is always equal to 360º

The formula is:

Sum of exterior angles of any polygon = 360º.



If you are working with a regular polygon:

Each **exterior angle** of any regular polygon of **n** sides is equal to the sum of the exterior angles divided by the number of sides.

The formula is:

$$\textbf{Each Exterior Angle} = \frac{\mathbf{360^\circ}}{\mathbf{n}}$$

Example: To find the measure of an exterior angle of a regular hexagon, which has 6 sides, apply the formula above as follows:

$$\text{Each Exterior Angle} = \frac{360^\circ}{n}$$

$$= \frac{360^\circ}{6}$$

$$= 60^\circ$$

Therefore, each exterior angle of a regular hexagon is 60º.

It is also possible to figure out how many sides a polygon has based on how many degrees are in its interior and exterior angles.

Example1

If each exterior angle measures 10º, how many sides does this polygon have?

Solution: Use the formula to find each exterior angle in reverse and solve for **n**.

$$\text{Each Exterior Angle} = \frac{360^\circ}{n}$$

$$10^\circ = \frac{360^\circ}{n}$$

$$10^\circ n = 360^\circ$$

$$n = \frac{360^\circ}{10^\circ}$$

$$n = 36$$

Therefore, the polygon has 36 sides.

Example 2

If each interior angle measures 140º, how many sides does this polygon have?

Solution: Use formula to find each interior angle in reverse and solve for **n**.

$$\text{Each Interior Angle} = \frac{(n-2)180^\circ}{n}$$

$$140^\circ = \frac{(n-2)180^\circ}{n}$$

$$140^\circ n = (n-2)180^\circ$$

$$140^\circ n = 180^\circ n - 360^\circ$$

$$180^\circ n - 140^\circ n = 360^\circ$$

$$40^\circ n = 360^\circ$$

$$n = \frac{360^\circ}{40^\circ}$$

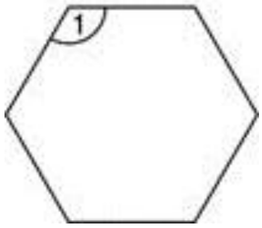
$$n = 9$$

Therefore, the polygon has 9 sides.

Sometimes you are asked to find the measure of each interior angle of a regular polygon given the number of sides. See example 3 on the next page.

Example 3

Find the measure of each interior angle of a regular hexagon (polygon with six sides).



Solution:

Method 1

Because the polygon is regular, all interior angles are equal, so you only need to find the interior angle sum and divide by the number of sides.

$$S = (6 - 2) \times 180^0$$

$$S = 4 \times 180^0$$

$$S = 720^0$$

There are six sides, so 720 ÷ 6 = 120°.

Each interior angle of a regular hexagon has a measure of 120°.

Method 2

Because the sum of the exterior angles of any polygons is 360° and since the polygon is regular, then each exterior angle would be 60°. (360° ÷ 6 = 60°)
And because an exterior and an interior angles form a linear pair, they are supplementary. So if the exterior angle is 60º, then the interior angle is 120º. (180º – 60º = 120º).

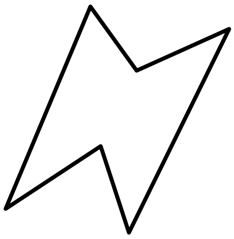
Each interior angle of a regular hexagon has a measure of 120°.

NOW DO PRACTICE EXERCISE 7

Practice Exercise 7

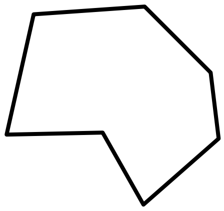
1. Write the name of each polygon in the space provided. (Do **not** include in your answer whether they are concave or convex).

(a)



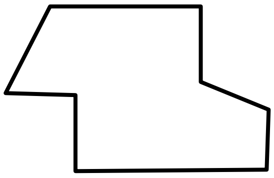
____

(b)



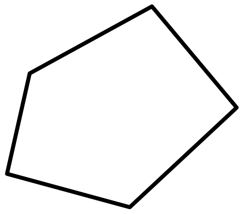
____

(c)



____

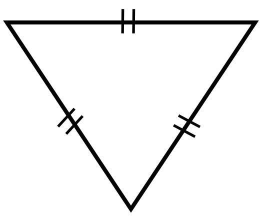
(d)



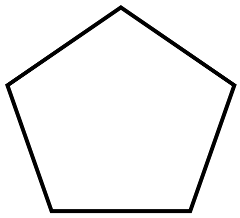
____

2. Measure the sides and the angles of these shapes and then answer the question below.

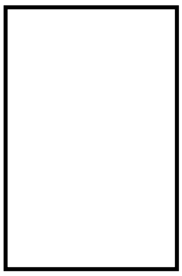
(a)



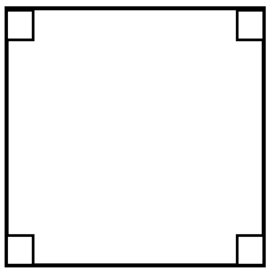
(b)



(c)



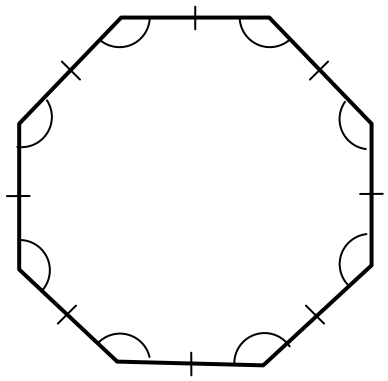
(d)



Which of the four polygons are regular polygons?

Answer: ______________

3. State whether the polygon is best described as equiangular, equilateral, regular, or none of these.



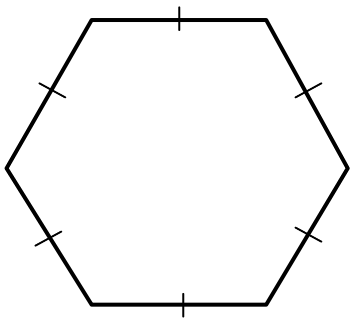
---

4. Describe the properties of: (a) heptagon (b) dodecagon.

	(a) heptagon	(b) dodecagon
Number of Sides	___________	___________
Exterior Angle Measure	___________	___________
Interior Angle Measure	___________	___________
Sum of Interior Angles	___________	___________

---

5. Identify the polygon and describe the properties.

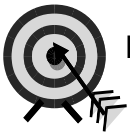


Polygon ___________
Exterior Angle Measure ___________
Interior Angle Measure ___________
Sum of Interior Angles ___________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4

Lesson 8: Triangles

You learnt about the different types of polygons in the previous lesson. You also learnt the formulas for finding interior and exterior angles of polygons and used them to work out problems.



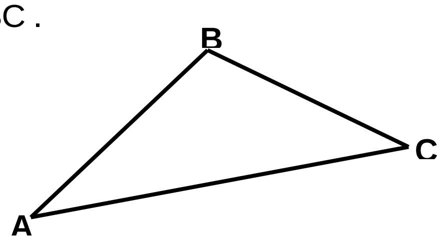
In this lesson you will:

- identify a triangle and its properties
- classify triangles
- find the angle sum of a triangle
- find the exterior angle of a triangle.

First, let us define a triangle.

A triangle is a three-sided figure with three angles in its interior. The symbol for triangle is Δ. A triangle is named by the three capital letters at its vertices (the plural for vertex), a fancy name for corners.

The figure below is ΔABC.



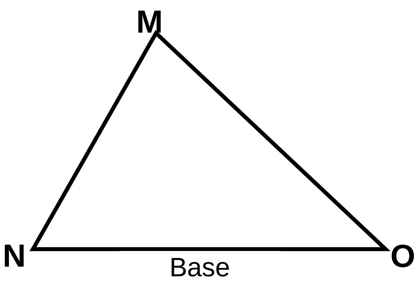
The primary parts of a triangle

1. The three sides
2. The three angles which always add up to 180º.
3. The three vertices

The secondary parts of a triangle

1. The Base

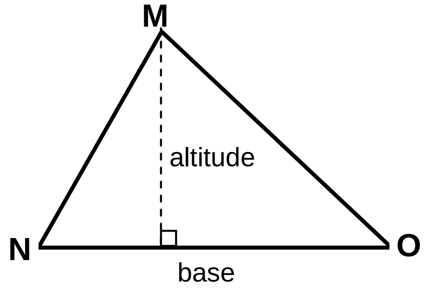
The **base of a triangle** can be any one of the three sides, usually drawn at the bottom.



You can pick any side you like to be the base. The base is commonly used as a reference side for calculating the area of a triangle.

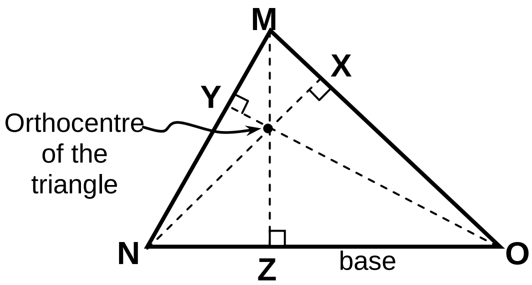
2. The Altitude

The **Altitude of a triangle** is the perpendicular line segment from the base to the opposite vertex.



In the figure, above, you can see one possible base and its corresponding altitude displayed.

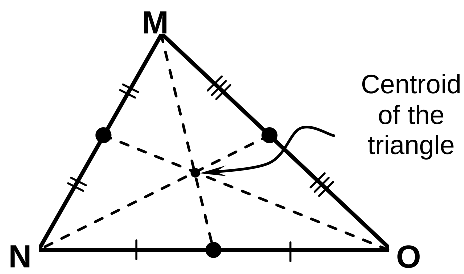
Since there are three possible bases, there are also three possible altitudes. The three altitudes intersect at a single point, called the **orthocentre** of the triangle. See figure below.



$\overline{MZ}$, $\overline{OY}$ and $\overline{NX}$ are altitudes

3. The Median

The **median of a triangle** is a line from a vertex to the midpoint of opposite side.



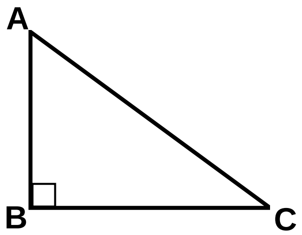
Triangles are classified according to their angles and according to their sides. All of them may be different from each other or the same size; any two sides or angles may be of the same size; there may be one distinctive angle.

The types of triangles classified by their **angles** include the following:

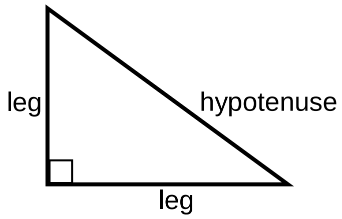
1. **Right Triangle**

A **right triangle** is a triangle with a right angle (i.e. 90°).

The ΔABC below is a right triangle. $\angle B$ is a right angle.



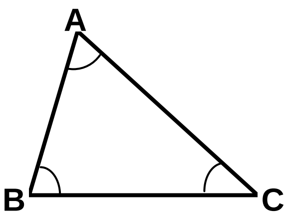
You may have noticed that the side opposite the right angle is always the triangle's longest side. It is called the **hypotenuse** of the triangle. The other two sides are called the **legs**.



2. **Acute Triangles**

An **acute triangle** is a triangle having all angles acute (less than 90°) in its interior.

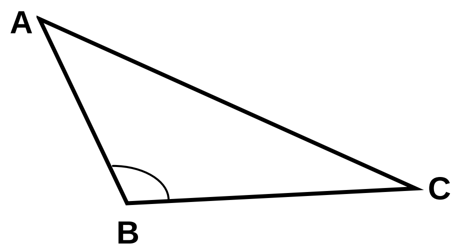
In the figure below, $\angle A$, $\angle B$ and $\angle C$ are all acute angles. ΔABC is an acute triangle.



3. **Obtuse Triangles**

An **obtuse triangle** is a triangle having an obtuse angle (greater than 90º but less than 180º) in its interior.

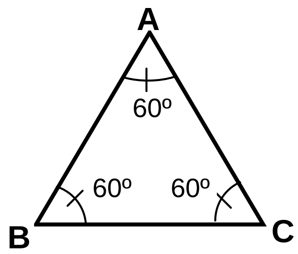
In the figure shown below, ΔABC is an obtuse triangle. $\angle B$ is an obtuse angle. The longest side of the triangle is always opposite the obtuse angle.



2. Equiangular Triangle

An **equiangular triangle** is a triangle having all angles of equal measure. The measure of the angles is always 60º.

In the figure shown below, ΔABC is an equiangular triangle.



Because the sum of all the angles of a triangle is 180º, the following theorem is easily shown.

Theorem: Each angle of an equiangular triangle has a measure of 60º.

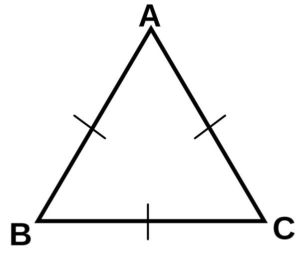
Triangles Classified by their sides

Besides classifying types of triangles according to the size of its angles as above: right triangles, acute triangles obtuse triangles and equiangular triangles; types of triangles can also be classified according to the length of its sides. Some examples are equilateral triangles, isosceles triangles and scalene triangles.

1. **Equilateral Triangles**

An **equilateral triangle** is a triangle with all three sides equal in measure.

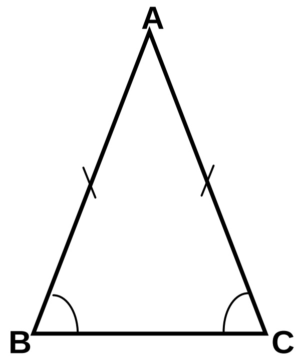
In the figure below, the slash marks indicate equal measure. $\overline{AB} = \overline{AC} = \overline{BC}$.
ΔABC is an equilateral triangle.



2. **Isosceles Triangles**

An **isosceles triangle** is a triangle with two sides equal in length.

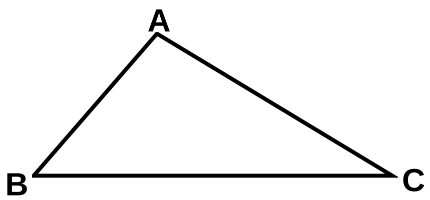
In an isosceles triangle, the two equal sides are called the **legs**. The third angle is called the **base**. The angles adjacent to the base are called the **base angles**. The angle opposite the base is called the **vertex angle**.



In the figure, ΔABC is an isosceles triangle, $\overline{AB} = \overline{AC}$ and are the **legs** of the triangle; $\overline{BC}$ is the base. $\angle B$ and $\angle C$ are the base angles. $\angle A$ is the vertex angle.

3. **Scalene Triangles**

A **scalene triangle** is a triangle with no sides of equal length.
Its angles are also all different in size.



In the figure, ΔABC is a scalene triangle, $\overline{AB} \neq \overline{BC} \neq \overline{AC}$. Also, $\angle A \neq \angle B \neq \angle C$.

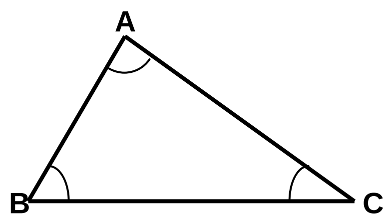
So far, we have learnt about the different types of triangle and their properties.. Now we will look at the angle properties of triangles.

Angle Sum Property

The sum of the interior angles of a triangle is equal to 180º.

The formula is:

$$\angle A + \angle B + \angle C = 180°.$$



Example:

If $\angle\mathbf{A} = 40°$ and $\angle\mathbf{B} = 60°$, find $\angle\mathbf{C}$.

Solution:

Because $\angle A \ \angle B + \angle C = 180°$

Then $40° + 60° + \angle C = 180°$.

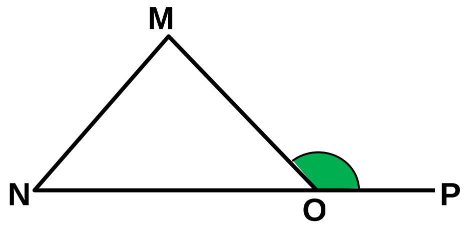
$\angle C = 180° – 40° – 60°$

$\angle C = 80°$.

Exterior angle of a Triangle

An exterior angle of a triangle is an angle formed when any side of a triangle is extended outwards.

For example, $\angle\mathbf{MOP}$ or $\angle\mathbf{4}$ is an exterior angle as the side **NO** of $\Delta\mathbf{MNO}$ is extended outwards to **P**.



Clearly, the exterior angle **MOP** and the adjacent interior angle **MON** are **supplementary.** That is**:**

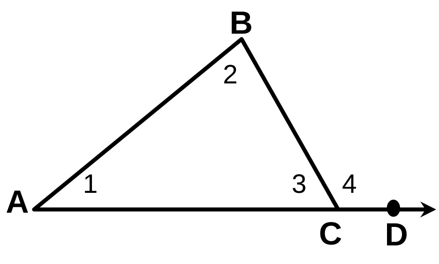
$$\angle\mathbf{MOP} + \angle\mathbf{MON} = 180^o$$

We can use equations to represent the measures of the angles described above. One equation might tell us the **sum of the angles** of the triangle.

For example, in the figure at the right

$$\angle 1 + \angle 2 + \angle 3 = 180^o$$

We know this is true, because the sum of the interior angles of a triangle is always 180^o. What is $\angle 4$? We do not know yet.



But because $\angle 4$ is an exterior angle of the triangle, and the adjacent interior angle is $\angle 3$. We say they are supplementary and equals to 180^o.

So, we can make a new equation: $\angle 3 + \angle 4 = 180^o$

If we combine the two equations, we can find that the measure of $\angle 4 = \angle 1 + \angle 2$.

(See next page how to do it).

Here is how to do it:

$$\angle 1 + \angle 2 + \angle 3 = 180^{\circ} \text{ (first equation)}$$
$$\angle 3 + \angle 4 = 180^{\circ} \text{ (second equation)}$$

Rewrite the second equation as $\angle 3 = 180 - \angle 4$ and substitute that for $\angle 3$ in the first equation:

$$\angle 1 + \angle 2 + (180 - \angle 4) = 180$$
$$\angle 1 + \angle 2 - \angle 4 = 0$$
$$\angle 1 + \angle 2 = \angle 4$$

The above equation tells us that the measure of the exterior angle of a triangle equals to the sum of the two interior angles. In fact, this is the **Exterior Angle Property or Theorem.**

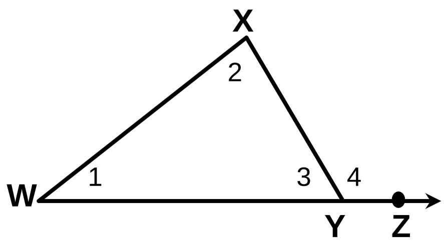
Exterior Angle Property

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote (non-adjacent) interior angles.

Now look at the examples.

Example 1

In the Figure at the right, if $\angle 1 = 30°$ and $\angle 2 = 100°$, find $\angle 4$.



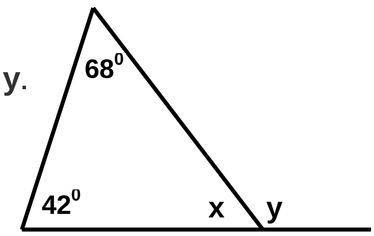
Solution: Because $\angle 4$ is an exterior angle of the triangle, using the Exterior Angle Property or Theorem, we have:

$$\angle 4 = \angle 1 + \angle 2$$
$$\angle 4 = 30° + 100°$$
$$\angle 4 = 130°$$

Therefore, the measure of $\angle 4$ is **130**°.

Example 2

In the figure at the right, find the measure of angles **x** and **y**.



Solution:

Since the three angles of a triangle add up to 180^0.

$$x + 42^0 + 68^0 = 180^0$$
$$x = 180^0 - 42^0 - 68^0$$
$$x = 70^0$$

Hence, the angle **x** is **70^0.**

The exterior angle of a triangle is equal to the sum of the opposite interior angles.

Hence,

$$y = 42^o + 68^o$$
$$= 110^o.$$

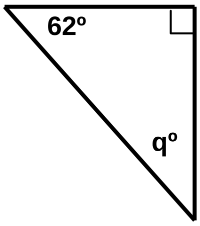
Therefore, the angle **y** is **110^o.**

NOW DO PRACTICE EXERCISE 8

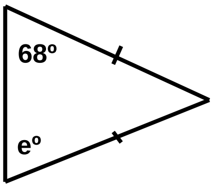
Practice Exercise 8

1. Find the value of pronumerals.

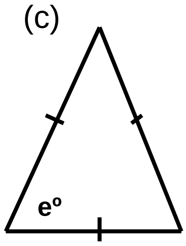
(a)



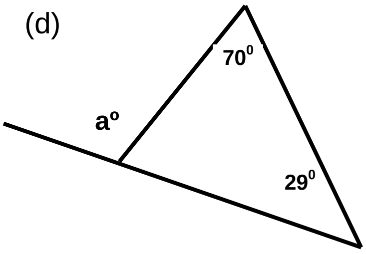
(b)



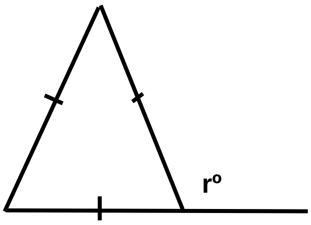
(c)



(d)

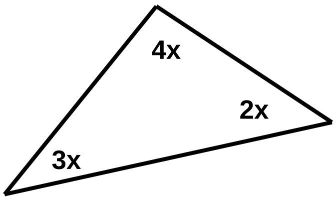


(e)

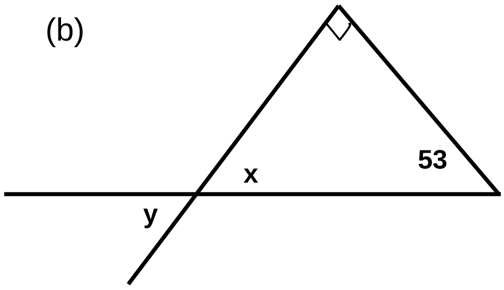


2 Find the value of x and y in these triangles

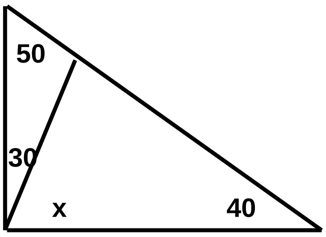
(a)



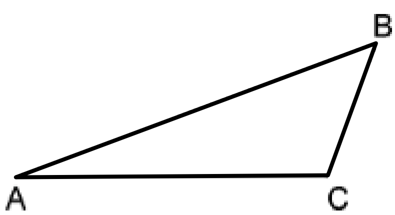
(b)



(c)



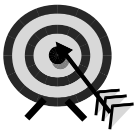
3. m∠A = 23, m∠B = 57. Find m∠C.



CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4

Lesson 9: Quadrilaterals

In the previous lesson, we discussed in detail one very important polygon which is the triangle.



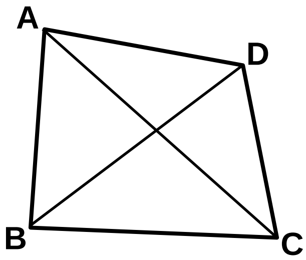
In this lesson, you will:

- define quadrilaterals
- identify the properties of quadrilaterals
- find the angle sum of a quadrilateral
- find the exterior angle of a quadrilateral.

The term "quadrilateral" was derived from the word "quad" that means "four" and the word "lateral" that means "sides". Thus,

A **quadrilateral is a polygon of four sides.**

For example, the plane figure bounded by four line segments **AB, BC, CD** and **DA** is called a quadrilateral and written as quadrilateral **ABCD.**



The sides: $\overline{\mathbf{AB}}$, $\overline{\mathbf{BC}}$, $\overline{\mathbf{CD}}$, and $\overline{\mathbf{DA}}$

The vertices: Points **A, B, C** and **D**

The angles: $\angle\mathbf{A}$, $\angle\mathbf{B}$, $\angle\mathbf{C}$ and $\angle\mathbf{D}$

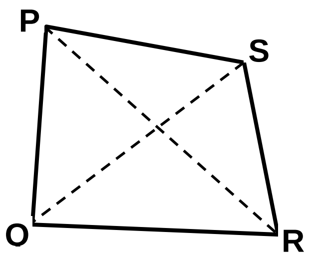
The diagonals: $\overline{\mathbf{AC}}$ and $\overline{\mathbf{BD}}$

There are two types of quadrilaterals on the basis of the diagonals.

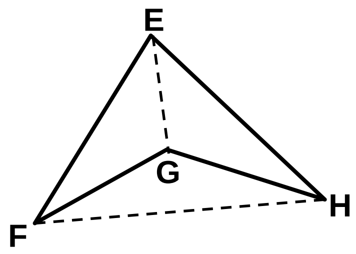
1. Convex
2. Concave

- **A convex quadrilateral** is a quadrilateral wherein the diagonals intersect in the **interior region.**
- **A concave quadrilateral** is a quadrilateral wherein one of the diagonals is in the **exterior region**.

See figures on the next page.



Convex quadrilateral

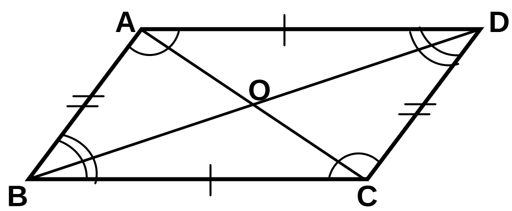


Concave quadrilateral

There are different properties of quadrilaterals. We can recognize the different types of quadrilateral on the basis of these properties.

1. **PARALLELOGRAM**

A parallelogram is a quadrilateral with two pairs of opposite sides equal and parallel.



Properties:

(a) The opposite sides parallel. $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$

(b) The opposite sides are equal. $\overline{AB} = \overline{DC}$ and $\overline{AD} = \overline{BC}$

(c) The opposite angles are equal. $\angle A = \angle C$ and $\angle B = \angle D$

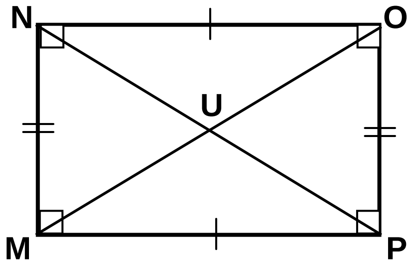
(d) The diagonals bisect each other. $\overline{AO} = \overline{OC}$ and $\overline{BO} = \overline{OD}$

(e) The adjacent angles are supplementary. $\angle A + \angle B = 180^o$, $\angle B + \angle C = 180^o$
$\angle C + \angle D = 180^o$ and $\angle D + \angle A = 180^o$

(f) It has no line of symmetry.

2. **RECTANGLE**

A rectangle is a parallelogram in which each angle is 90º.



□ means "right angle"
| and || show equal sides

Properties:

(a) The opposite sides parallel. $\overline{\mathbf{NM}} \parallel \overline{\mathbf{OP}}$ and $\overline{\mathbf{NO}} \parallel \overline{\mathbf{MP}}$

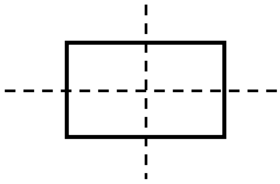
(b) The opposite sides are equal. $\overline{\mathbf{NM}} = \overline{\mathbf{OP}}$ and $\overline{\mathbf{NO}} = \overline{\mathbf{MP}}$

(c) Its angles are all right angles. $\angle\mathbf{M} = \angle\mathbf{N} = \angle\mathbf{O} = \angle\mathbf{P} = \mathbf{90^o}$.

(d) The diagonals are equal. $\overline{\mathbf{NP}} = \overline{\mathbf{MO}}$

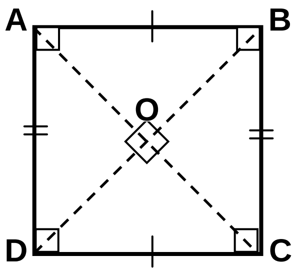
(e) The diagonals bisect each other. $\overline{\mathbf{NU}} = \overline{\mathbf{UP}}$ and $\overline{\mathbf{MU}} = \overline{\mathbf{UO}}$

(f) It has two lines of symmetry.



3. SQUARE

A square is a parallelogram with all sides equal and all angles are 90º.



□ means "right angle"

| and || show equal sides

Properties:

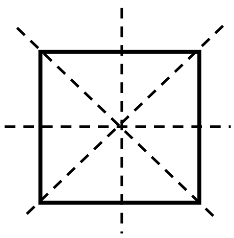
(a) All sides are equal. $\overline{\mathbf{AB}} = \overline{\mathbf{BC}} = \overline{\mathbf{CD}} = \overline{\mathbf{DA}}$

(b) Each angle is a right angle. $\angle\mathbf{A} = \angle\mathbf{B} = \angle\mathbf{C} = \angle\mathbf{D} = 90^o$

(c) Diagonals bisect each other at right angle.

(d) Diagonals are equal. $\overline{\mathbf{AC}} = \overline{\mathbf{BD}}$

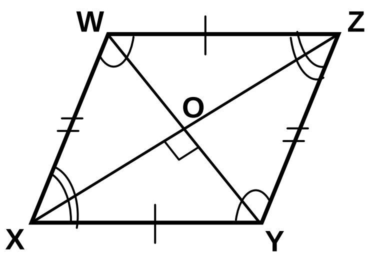
(e) It has four lines of symmetry.



A square is a special case of a rectangle in which all the sides are equal.

4. RHOMBUS

A rhombus is a parallelogram with all sides equal and parallel.



Properties:

(a) All sides are equal. $\overline{WX} = \overline{XY} = \overline{YZ} = \overline{ZX}$

(b) Opposite sides are parallel. $\overline{WX} \parallel \overline{ZY}$ and $\overline{WZ} \parallel \overline{XY}$

(c) Opposite angles are equal. $\angle W = \angle Y$ and $\angle X = \angle Z$

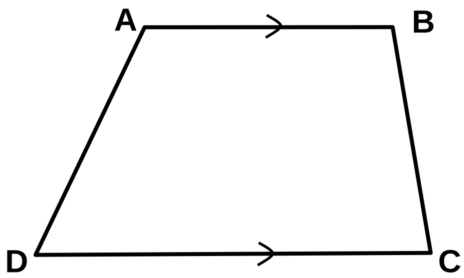
(d) Adjacent angles are supplementary. $\angle W + \angle X = 180^o$, $\angle Y + \angle Z = 180^o$

$\angle W + \angle Z = 180^o$ and $\angle X + \angle Y = 180^o$

(e) Diagonals bisect each other at right angle.

3. TRAPEZOID

A trapezoid is a quadrilateral with only one pair of opposite sides parallel. It is also called a trapezium.

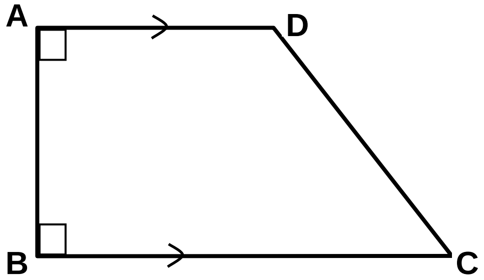


Properties:

(a) A pair of opposite sides parallel and are called the bases.
$\overline{AB} \parallel \overline{DC}$. $\overline{AB}$ is the upper base and $\overline{DC}$ is the lower base.

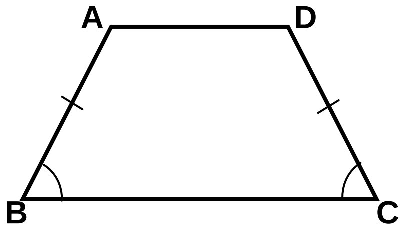
(b) The non – parallel sides are called the legs. $\overline{AD}$ and $\overline{BC}$ are legs.

A trapezoid with two right angles is called a **right trapezoid**. The figure below is a right trapezoid.



$\angle A$ and $\angle B$ are right angles.

If the two non-parallel sides or legs of a trapezoid are equal the trapezoid is called **isosceles trapezoid.** Below is an isosceles trapezoid.



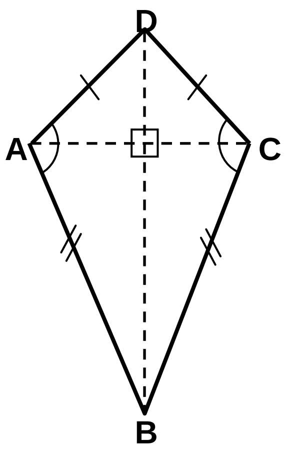
An isosceles trapezoid has the following properties:

(a) The two non-parallel sides (legs) are equal in length. $\overline{AB} = \overline{DC}$

(b) The base angles are equal. $\angle B = \angle C$

4. KITE

A **kite** is a quadrilateral that has two pairs of equal adjacent sides and unequal opposite sides.



Properties:

(a) Two pairs of adjacent sides equal. $\overline{AD} = \overline{DC}$

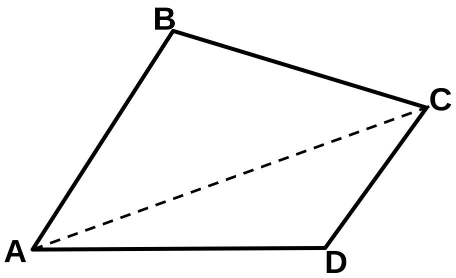
(b) One pair of opposite angles equal. $\angle A = \angle C$

(c) Longer diagonal bisects the shorter diagonal at right angle.

(d) It has one line of symmetry.

Now that you know the different types of quadrilaterals and their properties, let us proceed to the angle sum of quadrilaterals.

First let us draw a quadrilateral ABCD. Then draw the diagonal AC



Since the sum of the angles of each triangle is 180º, we get:

Angle sum of a Quadrilateral = 2(180º)

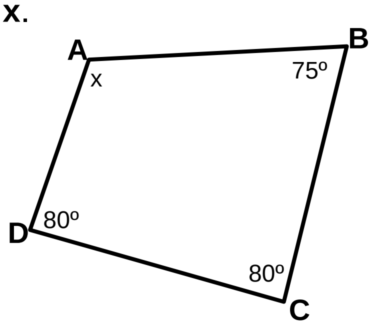
= 360º

The sum of all the angles of a quadrilateral is 360º.

It is often possible to find a missing angle when given some of the angles of a quadrilateral. It is important to remember that the sum of the angles of any quadrilateral is 360º.

Example 1

In the quadrilateral ABCD below, if $\angle$**B** = 75º, $\angle$**C** = 80º and $\angle$**D** = 80º, find the measure of the $\angle$**A** marked **x**.



We know that the angle sum of the quadrilateral is 360º.
So,

$\angle$**A** + $\angle$**B** + $\angle$**C** + $\angle$**D** = 360º

x + 75º + 80º + 80º = 360º

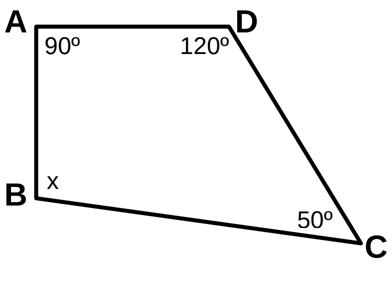
x + 235º = 360º (Add the known angles)

x = 360º – 235º (Subtract **235** from both sides)

x = 125º

Example 2

Find the value of the variable x in the diagram below.



Solution:

$\angle$**A** + $\angle$**B** + $\angle$**C** + $\angle$**D** = 360º

90º + x + 50º + 120º = 360º

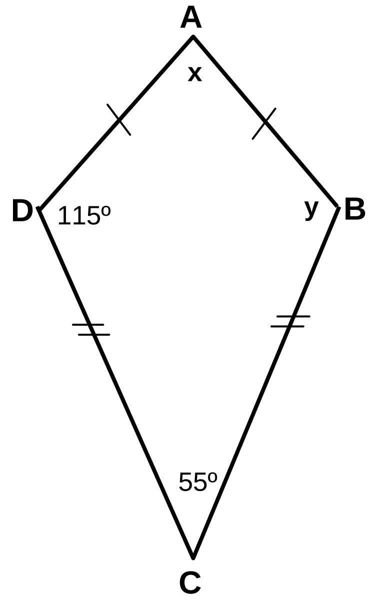
260º + x = 360º (Add the known angles)

x = 360º – 260º (Subtract **260** from both sides)

x = 100º

Example 3

Find the value of **x** and **y** in kite ABCD below.



Solution:

$\angle B$ and $\angle D$ are angles between two unequal sides of the kite ABCD.

$\angle B = \angle D$ (Angles between unequal angles of a kite are equal in size)

Since $\angle B = y$ and $\angle D = 115^o$, then **$y = 115^o$**.

Now, $x + y + 115^o + 55^o = 360^o$ (Angle sum of quadrilateral is 360º)

$x + 115^o + 115^o + 55^o = 360^o$

$x + 285^o = 360^o$ (Add all known angles)

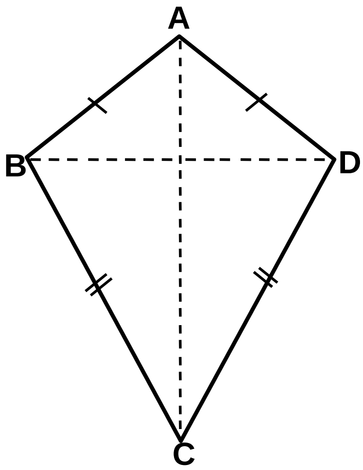
$x = 360^o - 285^o$ (Subtract 285 on both sides)

$x = 75^o$

NOW DO PRACTICE EXERCISE 9

Practice Exercise 9

1. Refer to the quadrilateral and answer the questions that follow.



(a) Name the sides of the quadrilateral.

________ , ________ , ________ , ________

(b) Name the two diagonals of the quadrilateral.

________ , ________

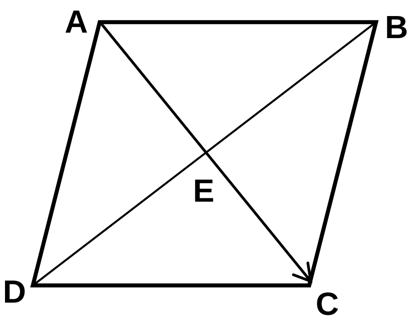
(c) Name the angles of the quadrilateral.

________ , ________ , ________ , ________

(d) Name the quadrilateral. ________.

---

2 The figure below shows a rhombus

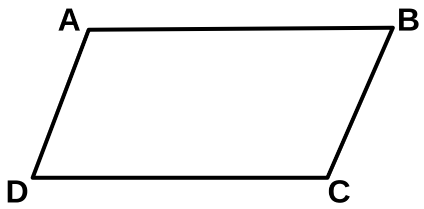


(a) Are triangles ABE and DEC congruent? (congruent means similar in shape and equal in size) ? __________

(b) Does $\angle$DAC equal $\angle$DCA? __________

(c) Is $\angle$DAB bisected by AC? __________

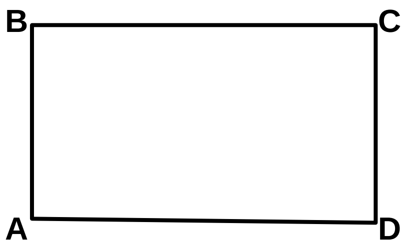
3. Which properties of parallelogram ABCD below are true? (Underline the letter of your answer).



(a) Sides AB and DC are parallel.
(b) Angles B and C are supplementary.
(c) Angle A equals Angle C
(d) The sum of all the angles is 360 degrees.
(e) all of the above.

---

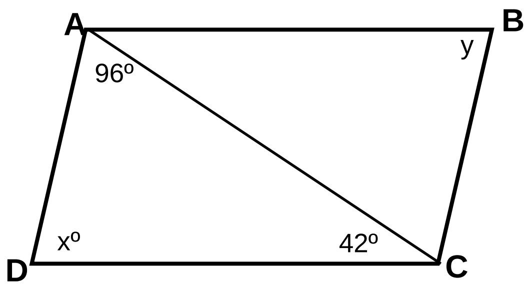
4. Rectangle ABCD below is a special kind of parallelogram because: (Underline the letter of your answer)



(a) It has all of the properties of a parallelogram
(b) All of its angles are 90 degrees in measure
(c) Its diagonals are congruent in length.
(d) All of the above

---

5. The figure below shows a parallelogram ABCD.



Find:

(a) The size of the angle marked x. ___________

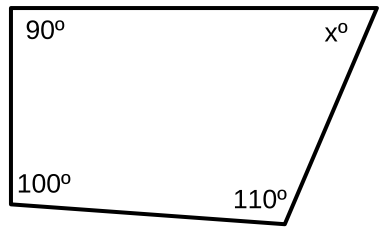
(b) The size of the angle marked y. ___________

(c) The size of $\angle$ACB ___________

(d) What type of triangle is ACD? ___________

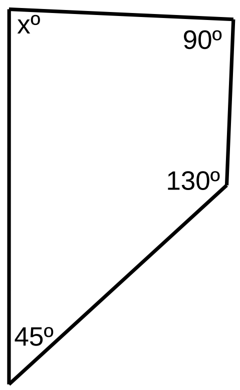
6.. Work out the value of $\angle$**x** in each quadrilateral.

(a)



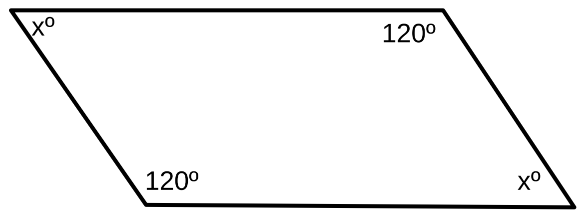
Answer: __________

(b)



Answer: __________

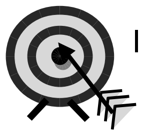
(c)



Answer: __________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

Lesson 10: Mixed Problems on Polygons



You learnt about the different types and properties of quadrilaterals in the previous lesson.

In this lesson, you will:

- solve mixed problems involving quadrilaterals by using their appropriate properties.

Each example problem that follows uses the steps in solving word problems and the properties of quadrilaterals. Use these problems as a guide in solving quadrilateral word problems.

Example 1

An isosceles trapezoid has base angles of 120°. What is the measure in each of the other angles of the trapezoid?

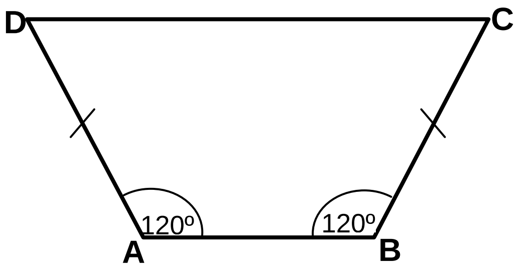
Solution:

Step 1: Read and understand the question.

This question is looking for the measure of each of the unknown angles in an isosceles trapezoid. The measures of the base angles are given.

Step 2: Make a plan.

Use the problem solving strategy of drawing a picture to help with this question. In an isosceles trapezoid, the two legs are congruent and the base angles are congruent. The following figure represents this trapezoid.



Step 3: Carry out the plan.

The base angles of the trapezoid are congruent, so $\angle A = 120°$ and $\angle B = 120°$.

To find the measure of the other angles, subtract the sum of the base angles from 360°. $360 - 240 = 120$,

Divide 120 by 2 $\dfrac{120}{2} = 60$

Therefore, the measure of each of the other angles is **60°**.

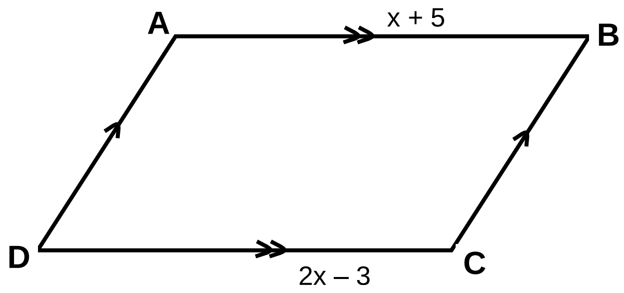
Thus, the angles measure 120°, 120°, 60°, and 60°, respectively.

Step 4: Check your answer.

To check this answer, add the measures of the four angles to be sure that the total number of degrees is 360: 120 + 120 + 60 + 60 = 360°

Example 2

A parallelogram has two opposite sides labeled $x + 5$ units and $2x – 3$ units, respectively. What is the length of these opposite sides?



Solution:

Step 1: Read and understand the question.

This question is looking for the length of each of the opposite sides in a parallelogram.

Step 2: Make a plan.

The lengths of opposite sides of a parallelogram are equal. Set the given expressions equal to each other, and solve for x. Then, substitute the value of x into one of the expressions to find the length of the sides.

Step 3: Carry out the plan.

Set the expressions equal to each other: $\overline{AB} = \overline{DC}$ $x + 5 = 2x – 3$.

Subtract x from each side of the equation to get $5 = x – 3$.

Add 3 to each side of the equation to get $8 = x$.

Substitute x = 8 into the expression x + 5. $8 + 5 = 13$.

The length of each of the opposite sides is **13 units**.

Step 4: Check your answer.

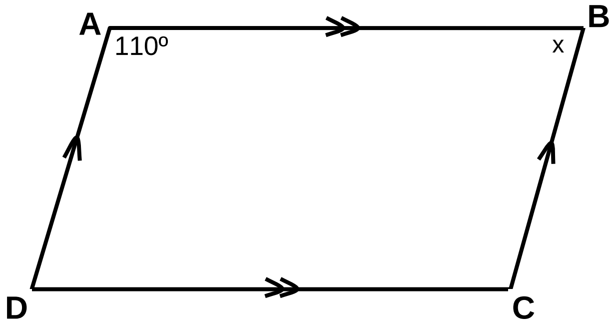
To check this answer, substitute $x = 8$ into the expression $2x – 3$ to be sure the value is also 13.

$$2(8) – 3 = 16 – 3 = 13$$

Therefore, $\overline{AB} = 13$ units and $\overline{DC}$ = 13 units.

Example 3

Here is another parallelogram. Use its properties to work out the value of $\angle$**x**:



Solution:

Opposite angles of parallelogram are equal. So $\angle A = \angle C = 110^o$
$\angle B = \angle D = x$

To find value of x, use angle sum property. $\angle A + \angle B + \angle C + \angle D = 360^o$
$110^o + 110^o + x + x = 360^o$

Add like terms $220 + 2\mathbf{x} = 360$

Subtract **220** from both sides $2\mathbf{x} = 360 - 220$

Divide both sides by **2** $2\mathbf{x} = 140$

$\mathbf{x = 70^o}$

An alternative method of finding angle **x** is to use the fact that angle **x** and 110º are adjacent angles.

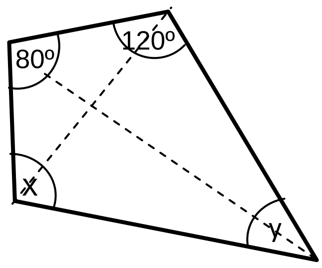
Adjacent angles of parallelogram are supplementary. That is $\mathbf{x} + 110^o = 180^o$

Subtract **110** from both sides $\mathbf{x} = 180^o - 110^o$

$\mathbf{x = 70^o}$

Example 4

The kite at the right has two unknown angles, **x** and **y**.



Solution:

To find the value of unknown angles, the properties required are:

(a) The angle sum of a quadrilateral is 360º.

(b) The kite is symmetrical about its long diagonal.

Using the property (b)

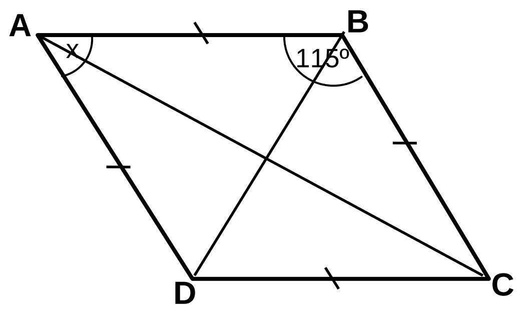
The value of **x** is 120º.

We calculate the value of **y** using property (a).

For example:

$$80 + 120 + \mathbf{y} + \mathbf{x} = 360$$
$$80 + 120 + \mathbf{y} + 120 = 360$$
$$\mathbf{y} + 320 = 360$$
$$\mathbf{y} = 360 - 320$$
$$\mathbf{y} = \mathbf{40^o}$$

Example 5



ABCD is a rhombus.

Solve for $\angle$**x**.

Solution:

The properties required are:

(a) the opposite angles are equal.
(b) the angle sum is 360º.

We are given the following information: $\angle$**B** = 115º and $\angle$**A** = **x**

Hence,

$\angle$**B** = $\angle$**D** =115º (Opposite angles are equal)
and $\angle$**A** = $\angle$**C** = **x** (Opposite angles are equal)

so $\angle A + \angle B + \angle C + \angle D = 360$ (Angle sum is **360º**)
$\mathbf{x} + 115 + \mathbf{x} + 115 = 360$
$230 + 2\mathbf{x} = 360$ (Add like terms)
$2\mathbf{x} = 360 - 230$ (Subtract **230** from both sides)
$2\mathbf{x} = 130$
$\mathbf{x} = \mathbf{65^o}$ (Divide both sides by **2**)

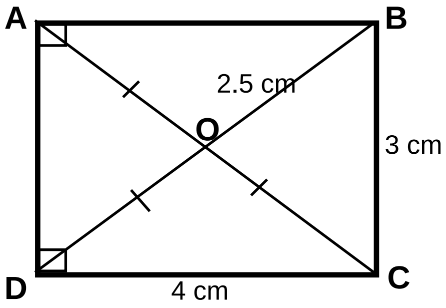
A quicker method is to use the fact that $\angle$**A** and $\angle$**B** are the co-interior angles

that is $\angle A + \angle B = 180^o$
therefore $\mathbf{x} + 115 = 180$ (Since **x** = $\angle$**A)**
so $x = 180 - 115$
$\mathbf{x} = \mathbf{65^o}$

NOW DO PRACTICE EXERCISE 10

Practice Exercise 10

1. Answer the questions below by looking at this parallelogram.



Work out $\overline{\mathbf{AC}}$ by completing these statements.

(a) The diagonals _______________ each other.

$\overline{\mathbf{DO}}$ = _____ cm $\overline{\mathbf{DO}}$ = $\overline{\mathbf{BO}}$ = _____ cm $\overline{\mathbf{BD}}$ = _____ cm

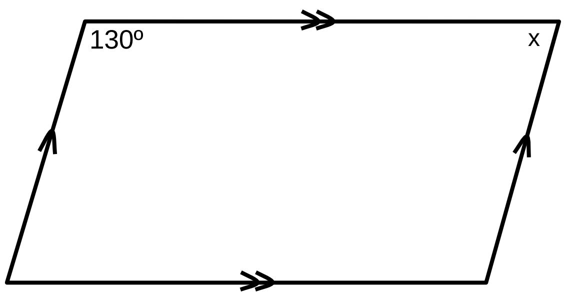
(b) The diagonals are _____________

$\overline{\mathbf{BD}}$ = _______ cm. Hence, $\overline{\mathbf{AC}}$ = _______ = ________ cm

(c) The parallelogram above is a _______________.

---

2. Work out the value of **x**.



Properties required are:

(a) The opposite angles are ____________.

(b) The angle sum of a parallelogram is ____________.

130 + **x** + 130 + **x** = _____

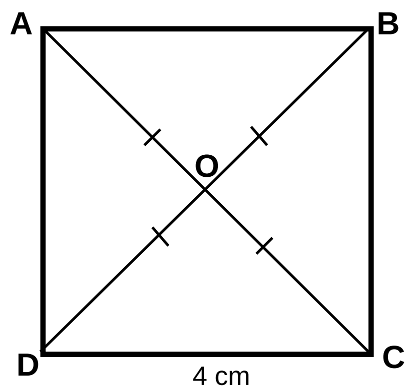
_____ + _____ = _____

______ = ______ − ______

______ = ______

x = ______

3.



(a) Name this special kind of parallelogram.

Answer: ____________

(b) If $\overline{\textbf{AC}}$ = 5.2cm, what is the length of $\overline{\textbf{BO}}$?

Answer: ______________

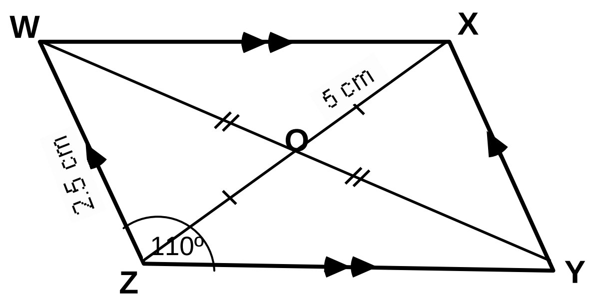
(c) $\overline{\textbf{BD}}$ bisects ∠**ABC**. Work out the size of ∠**OBC**.

Answer: ____________

---

4. Study this diagram of a parallelogram, and then find:

(a) $\overline{\textbf{OZ}}$,
(b) $\overline{\textbf{XY}}$
(c) ∠**XYZ**.



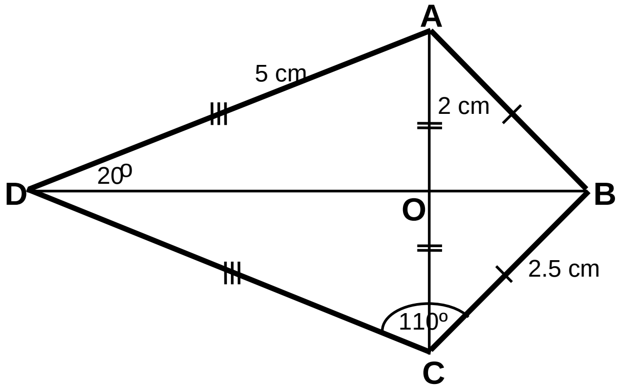
Properties required:

1. ____.
2. ____.
3. ____.
4. ____.

Answer:

(a) $\overline{\textbf{OZ}}$ = _____________
(b) $\overline{\textbf{XY}}$ = _____________
(c) ∠**XYZ** = ______________.

5. Use the properties of a kite to answer the questions below.



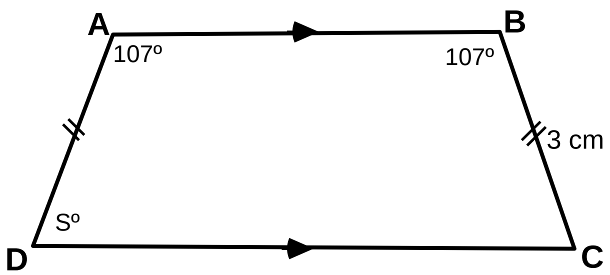
a. Name the symmetrical line. **Answer**: _______________

b. What is the length of **AC**? **Answer**: _______________

c. What is the size of ∠**DAB**? **Answer**: _______________

d. Calculate ∠**ABC**. **Answer**: _______________

6.



(a) Complete these statements:

ABCD is an ______________ trapezium.

BC = ______________. Hence, **AD** = __________ cm

(b) Calculate the value of **s**

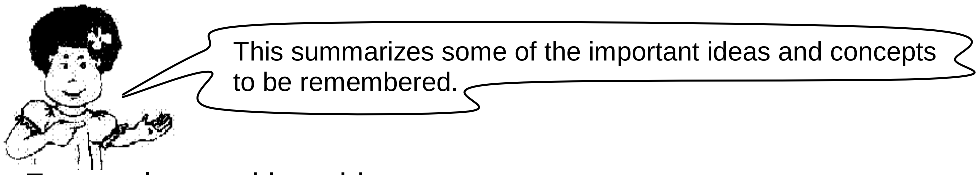
107º + **s** = _________ (co-interior angle)

s = ________ – ________

s = ________º.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

TOPIC 2: SUMMARY



- For a polygon with **n** sides:
  (a) The sum of the interior angles is (n – 2) x 180º.
  (b)The sum of the exterior angles is 360º.
  (c)Each exterior angle of a regular polygon is $\frac{360}{2}$.
  (d)Each interior angle of a regular polygon is $\frac{(n-2)x180^{\circ}}{n}$.
- **An equilateral triangle** is a triangle with three equal sides.
- **An isosceles triangle** is a triangle with two sides of equal length.
- **A scalene triangle** is a triangle with no sides of equal length and no equal angles.
- **An equiangular triangle** is a triangle with three equal angles.
- **A right triangle** is a triangle with a right angle.
- **An acute triangle** is a triangle having all angles acute.
- **An obtuse triangle** is a triangle having an obtuse angle in its interior.
- For a triangle:
  (a)The sum of the interior angles is 180º.
  (b)An **exterior angle** of a triangle is an angle formed when any side of the triangle is extended outwards.
  (c)The measure of the exterior angle of a triangle is equal to the sum of the two non-adjacent interior angles.
- **A quadrilateral** is a polygon of four sides.
- A quadrilateral wherein the diagonals intersect in the interior region is called **convex.**
- A quadrilateral wherein one of the diagonals is in the exterior region is called **concave.**
- A quadrilateral with two pairs of opposite sides equal and parallel is called **parallelogram.**
- A parallelogram in which each angle is 90º is called a r**ectangle**.
- A rectangle in which all the sides are equal is called a **square**.
- A parallelogram with all sides equal and parallel is called a **rhombus**.
- A quadrilateral with only one pair of opposite sides parallel is called a **trapezoid**.
- A trapezoid whose two non-parallel sides are equal is called an **isosceles trapezoid**.
- A quadrilateral that has two pairs of equal adjacent sides and unequal ooposite sides is called a **kite**.
- For any quadrilateral:
  (a)The sum of all the angles is 360º.

REVISE LESSONS 7- 10. THEN DO TOPIC TEST 2 IN ASSIGNMENT BOOK 5.

ANSWERS TO PRACTICE EXERCISES 7 TO 10

Practice exercise 7

1. (a) hexagon (b) heptagon (c) octagon (d) pentagon

2. a, b, and d

3. regular- all sides are equal and all angles are equal.

4. (a) Heptagon
   Number of sides = 7
   Exterior angle measure = 51.43º
   Interior angle measure = 129º
   Sum of interior angle = 900º

   (b) Dodecagon
   Number of sides = 12
   Exterior angle measure = 30º
   Interior angle measure = 150º
   Sum of interior angles = 1800º

5. Polygon = Regular hexagon
   Exterior Angle Measure = 60º
   Interior Angle Measure = 120º
   Sum of Interior Angles = 720º

Practice exercise 8

1. (a) 28º (b) 68º (c) 60º (d) 99º (e) 120º

2. (a) 20º (b) 37º (c) 60º

3. $m\angle C = 100^\circ$

Practice exercise 9

1. (a) Sides - $\overline{AB}$, $\overline{AD}$, $\overline{BC}$ and $\overline{DC}$
   (b) Diagonals- $\overline{BD}$ and $\overline{AC}$
   (c) $\angle A$, $\angle B$, $\angle C$, and $\angle D$
   (d) Kite

2. (a) Yes (b) Yes (c) Yes

3. (e) all of the above

4. (d) all of the above

5. (a) 42º (b) 42º (c) 96º (d) isosceles triangle

6. (a) 60º (b) 95º (c) 60º

Practice exercise 10

1. (a) The diagonals bisect each other. $\overline{\mathbf{DO}}$ = 2.5 cm $\overline{\mathbf{BO}}$ = 2.5 cm $\overline{\mathbf{BD}}$ = 5 cm
   (b) The diagonals are equal. $\overline{\mathbf{BD}}$ = 5 cm, $\overline{\mathbf{AC}}$ = $\overline{\mathbf{BD}}$ = 5 cm.
   (c) The parallelogram is a rectangle.

2. (a) The opposite angles are equal.
   (b) The angle sum of a parallelogram is 360º
   x = 50º

3. (a) square (b) 2.6 cm (c) 45º

4. (a) 5 cm (b) 2.5 cm (c) 70º

5. (a) $\overline{\mathbf{BD}}$ (b) 4 cm (c) 110º (d) 120º

6. (a) **ABCD** is an isosceles trapezium. $\overline{\mathbf{BC}}$ = 3 cm, $\overline{\mathbf{AD}}$ = 3 cm

   (b) 107º + s = 180º
   s = 180º – 107º
   s = 73º

END OF TOPIC 2

TOPIC 3

AREA

TOPIC 3: AREA

Introduction

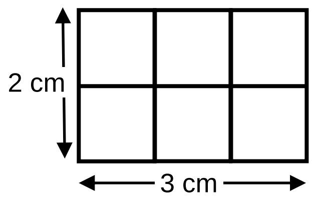
The area of a shape is the amount of surface it covers or occupies.

For example, a park occupies a certain amount of land surface.

Calculating areas is an important skill used by many people in their daily work. Builders and tradespeople often need to work out the areas and dimensions of the structures they are building, and so do architects, designers and engineers

We may measure area in square millimetres (mm^2), square centimetres (cm^2), square metres (m^2) or even square kilometres (km^2).

Area maybe thought of as the number of squares that would fit inside a shape. For example, 6 square centimetres fit exactly inside the rectangle below so its area is 6 cm^2.



You may recall the formula for finding the area of rectangle.

Area = length x width
Area = lw

So for the rectangle above, A = 3 x 2 = 6 cm^2

You may also recall the formula for the area of a square:

Area = side x side
Area = s^2

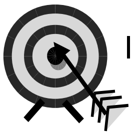
Using formula to find area saves counting squares, so it is convenient when we are dealing with large or non-rectangular shapes.

In this topic, you will extend your knowledge about the area of different plane figures and its application in real life situations.

.

Lesson 11: Area of Rectangles and Triangles

You learnt about the area of triangles and rectangles in Grade 7 Mathematics.



In this lesson, you will:

- derive the formula for finding the area of a rectangle and a square
- find the area of a rectangle and a square.
- derive the formula for finding the area of triangle
- find the area of a triangle

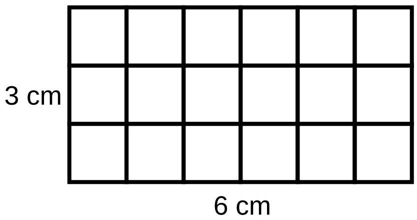
First let us revise the two ways of finding the area of a rectangle.

(1) By counting the number of 1 cm^2 squares.
(2) By using the formula: Area = length x width

For example, think of an carpet which covers a space. The LENGTH is the number of floor tiles covered in each row and the WIDTH is the number of rows of tiles. The AREA is the total number of *square* tiles covered.

Find the area.

.



Using the method of counting square, we find that the area of the rectangle is 18 cm^2

Clearly, the rectangle contains 3 rows of 6 squares. Therefore:

Area = 3 cm x 6 cm
= 18 cm^2

This suggests that:

The area of a rectangle is equal to its length multiplied by its width. That is:

Area = Length x Width

Using the letters **A** for area, **L** for length and **W** for width, we can write it simply as:

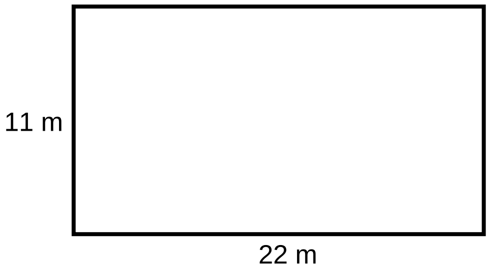
$$\boxed{\mathbf{A = LW}}$$

This is the formula for the area of a rectangle.

Example 2

Find the area of a rectangular field 22 m long and 11 m wide.

Solution:



A = LW

A = (22 m)(11 m)

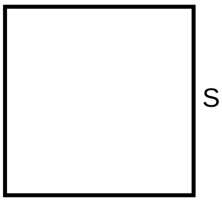
A = 242 m^2

So, the area of the field is **242 m^2.**

Area of a square

A **square** is a special rectangle with its length equal to its width. That is, all sides are **equal**. Since a square is also a rectangle the formula for area of a rectangle is used to calculate its area.

That is:



Area of a square = length x width,

but length = width, so **A** = side x side

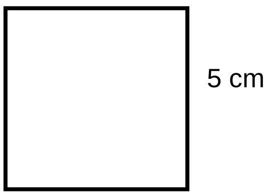
= $(\text{side})^2$

= s^2

Let us work out an example.

Example 3

Find the area of this square whose side is 5 cm.



Area = side x side .... **(s = 5 cm)**

= 5 x 5

= 25 cm^2

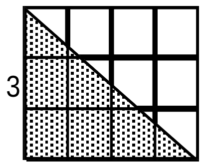
Now, go to the next page.

Area of a Triangle

We will now revise how the formula for the area of a triangle is obtained from a rectangle, and then use it to calculate the area of triangles.

Look at the right angle triangle (shaded) inside the rectangular grid below.

The length (4 units) of the rectangle is really the **base** of the triangle and the **width** (3 units) of the rectangle is the **height**.



Area of rectangle = Length **x** Width .... **L = 4 units, W = 3 units**

Area of triangle $= \frac{1}{2}$ the area of a rectangle

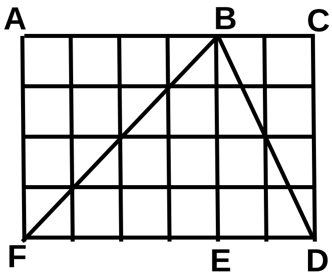
$= \frac{1}{2}$ **x base x height** .... **base = L = 4 units**

$= \frac{1}{2}$ **x** 4 **x** 3 sq. units **height = W = 3 units**

= **6 sq. units**

We will see how this is also true for other triangles.

For example: The figure opposite shows a non-right angle triangle drawn on a rectangular grid.



Area of rectangle **ACDF** = Length **x** Width ..... **L = 6 units, W = 4 units**

= 6 **x** 4 sq. units

= **24 sq units**.

Area of triangle **FBD** = Area of triangle **EBF** + Area of triangle **EBD**

$= \frac{1}{2}$ area of rectangle **ABEF** + $\frac{1}{2}$ area of rectangle **BCED**

$= \frac{1}{2}$ **x** 16 + $\frac{1}{2}$ **x** 8

= 8 + 4 sq. units

= 12 sq. units

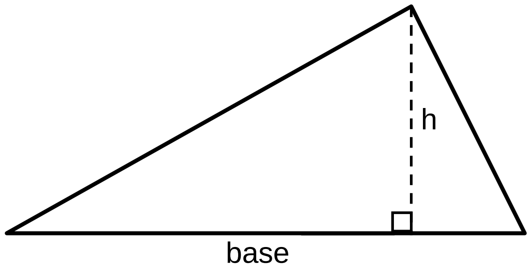
$\therefore$ Area of Triangle **FBD** = $\frac{1}{2}$ **x** Area of Rectangle **ACDF**

= $\frac{1}{2}$ **x FD x BE**

= $\frac{1}{2}$ **x** base **x** height

To find the area of a triangle, we must know its base and perpendicular height (**h**).

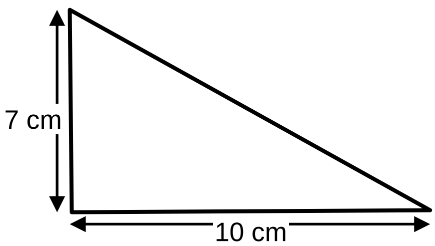
Look at this triangle.



Notice that the base and height are perpendicular to each other. That is, the height is at a right angle to the base.

Let us have a look at some examples on how to find the area of triangles using the formula.

Example 1



b = 8 cm, h = 7 cm

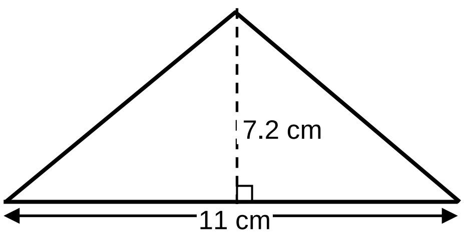
Area of triangle = $\frac{1}{2}$ **x** b **x** h

= $\frac{1}{2}$ **x** 10 **x** 7

= 5 **x** 7

= **35 cm^2**

Example 2



b = 11 cm, h = 7.2 cm

Area of triangle = $\frac{1}{2}$ **x** b **x** h

= $\frac{1}{2}$ **x** 11 **x** 7.2

= 5.5 **x** 7.2

= **39.6 cm^2**

Example 3

A flower garden is in the shape of a triangle whose base is 12.5 m and height 14 m. Find its area.

Solution:

Area of garden = $\frac{1}{2}$ **x** b **x** h .... b = 12.5 m, h = 14 m

= $\frac{1}{2}$ **x** 12.5 **x** 14

= **175 m^2**

So far, we have learned to calculate the area of a triangle. We will now learn to calculate the **base** of a triangle given its area and height.

Take the formula: $\mathbf{A} = \frac{1}{2}$ bh. Where: '**A**' represents area, '**b**' the base and '**h**' the height of the triangle. Now make '**b**' the subject of the formula.

$A = \frac{1}{2}bh$ (multiply both sides by **2**)

$2A = bh$ (divide both sides by h)

$\frac{2A}{h} = b$

$$\boxed{\mathbf{b} = \frac{\mathbf{2A}}{\mathbf{h}}}$$

Use this formula to find the base of a triangle.

Example 4

Find the base of a triangle with an area of 81 cm^2 and a height of 9 cm.

Solution: $b = \frac{2\mathbf{A}}{h}$

$= \frac{2 \times 81}{9}$

$= \mathbf{18\ cm}$

Now we will calculate the height of a triangle given its area and base.
This time make '**h**' the subject of the formula.

$A = \frac{1}{2} \times b \times h$ .... (multiply both sides by **2**)

$2A = b\,h$ .... (divide both sides by b)

$\frac{2A}{b} = h$

$$\boxed{\mathbf{h = \frac{2A}{b}}}$$

Use this formula to find the height of a triangle.

Example 5

Find the height of a triangle with an area of 48 cm^2 and a base of 8 cm.

Solution: $h = \frac{2A}{b}$

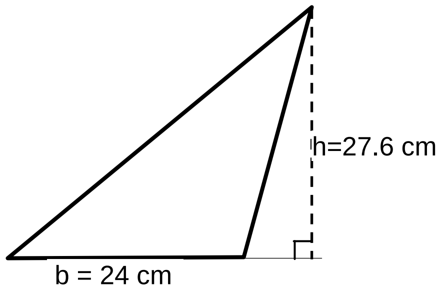
$= \frac{2 \times 48}{8}$

$\mathbf{h = 12\ cm}$

Sometimes you will encounter problems involving area of triangles wherein the height can actually extend outside the triangle. So technically, the height does not necessarily intersect with the base.

Example 6

What is the area of the triangle pictured below?



Solution:

To find the area of the triangle, substitute the base and the height into the formula for area.

$$A = \frac{1}{2}\ b\ h$$

$$= \frac{1}{2}(24)(27.6)$$

$$= \frac{1}{2}(662.4)$$

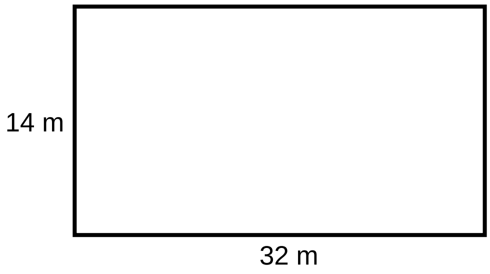
$$\mathbf{= 331.2\ cm^2}$$

NOW DO PRACTICE EXERCISE 11

Practice Exercise 11

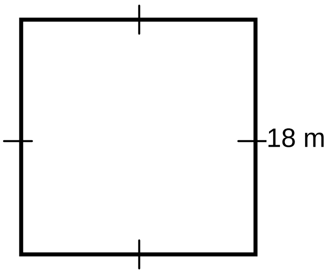
1. Calculate the area of the following shapes.

(a)



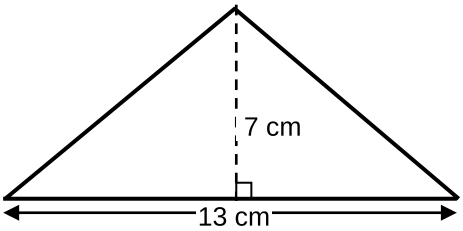
Answer: ______________

(b)



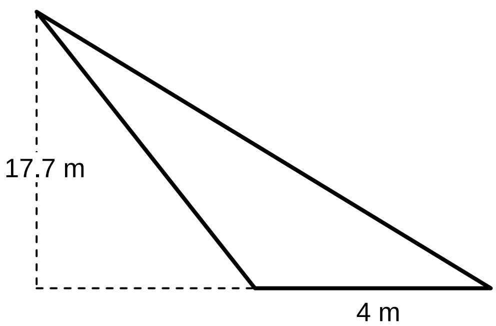
Answer: ______________

(c)



Answer: ______________

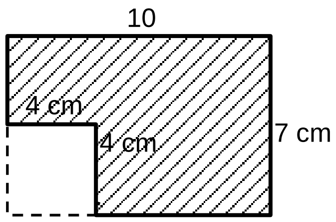
(d)



Answer: ______________

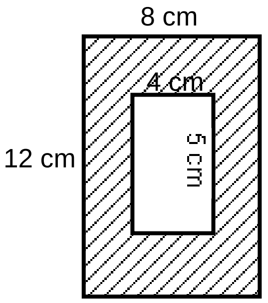
2. Calculate the shaded area in each of the following shapes.

(a)



Answer: ______________

(b)



Answer: ______________

---

3. Find the base or height of these triangles that have the following measurements:

(a) Area = 200 cm^2, height = 8 cm

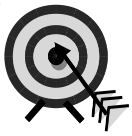
Answer: ______________

(b) Area = 20 cm^2, base = 8 cm

Answer: ______________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

Lesson 12: Area of Parallelogram, Rhombus and Trapezoid



You learnt how to work out areas of rectangle, square and triangles in the last lesson.

In this lesson, you will:

- derive the formula for finding the area of a parallelogram, rhombus or trapezoid
- calculate the area of parallelogram, rhombus or trapezoid.

We learnt that a parallelogram is a quadrilateral with two opposite sides equal and parallel.

Finding the area of a parallelogram means you need to count the number of unit squares inside the parallelogram. This is a bit more difficult than for the rectangle since the parallelogram cuts many of the unit squares into fractional pieces.

However, we can rearrange the parallelogram into a more familiar figure without changing its original area. Then we can find the area of the familiar figure, which will have the same area as the parallelogram.

Look at the figures.

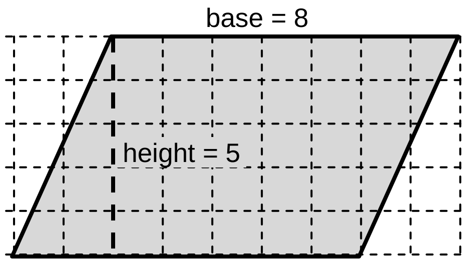


Figure 1

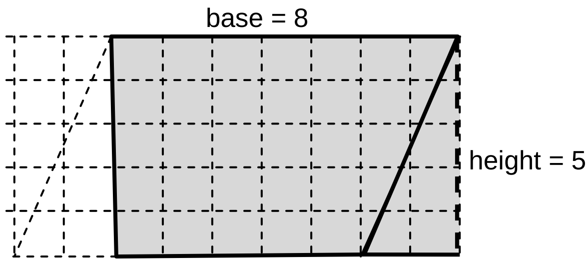


Figure 2

In Figure 1, you can cut the triangle from the left of the parallelogram and move it to the right side of the parallelogram, making a rectangle whose length is the base of the parallelogram and the width is the height of the parallelogram as shown in Figure 2.

We learnt that the formula for finding the rectangle's area is **base *x* height**, so the area of this shape is 8 x 5, which is 40 square units.

Basically, to find the area of a parallelogram you use the same formula as with rectangles.

Area = base (Length) x height (Width)

$$\boxed{\mathbf{A = b \times h}}$$

The base and height of the parallelogram must be perpendicular. However, the lateral sides of a parallelogram are not perpendicular to the base. Thus a dashed line is drawn to represent the height.

Let us look at some examples involving the area of a parallelogram.

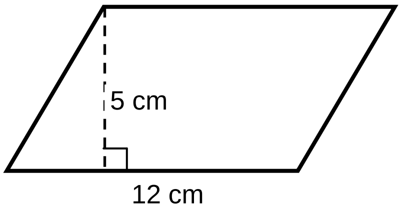
Example 1

Find the area of a parallelogram with a base of 12 cm and a height of 5 cm.

Solution: $A = b \times h$

$= (12\text{ cm})(5\text{ cm})$

$\mathbf{= 60\text{ cm}^2}$



Given the area of a parallelogram and either the base or the height, we can find the missing dimension using the following formula:

$\boxed{h = \frac{A}{b}}$ formula for height $\boxed{b = \frac{A}{h}}$ formula for base

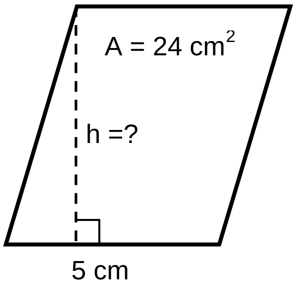
Example 2

The area of a parallelogram is 24 cm^2 and the base is 4 cm.
Find the height.

Solution: $h = \frac{A}{b}$

$h = \frac{24\text{ cm}^2}{4\text{ cm}}$

$h = 6$ cm



Example 3

The area of a parallelogram is 64 cm^2 and the height is 16 cm.
Find the base.

Solution: $b = \frac{A}{h}$

$b = \frac{64\text{ cm}^2}{16\text{ cm}}$

$b = 4$ cm

On the next page, you will learn how to derive the formula for finding the area of a trapezoid.

Area of a Trapezoid

The technique for finding the formula for the area of a trapezoid is exactly the same technique we used for the triangles.

Study the figures.

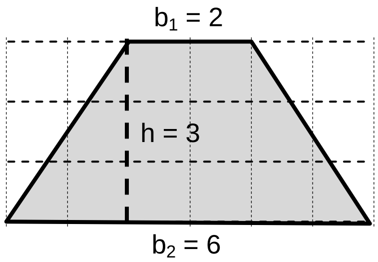


Figure 3

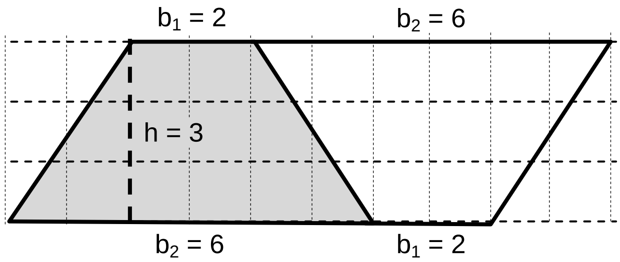


Figure 4

Figure 3 is doubled and arranged to create a familiar parallelogram as shown in Figure 4.

Notice that the base of the parallelogram formed in Figure 4 is the sum of the upper and lower bases of the trapezoid in Figure 3.

So if we find the area of the parallelogram, we have

$$\begin{aligned} \text{Area of parallelogram } &= (b_1 + b_2) \text{ x } h \\ &= (2 + 6) \text{ x } 3 \\ &= 8 \text{ x } 3 \\ &= 24 \text{ square units} \end{aligned}$$

However, we only want the area of the trapezoid, so we have to divide the area of the parallelogram by 2, thus we have

$$\begin{aligned} \text{Area of trapezoid } &= (\text{Area of parallelogram}) \div 2 \\ &= 24 \div 2 \\ &= 12 \text{ square units} \end{aligned}$$

To put this entire process on one line it would look like this:

Area of trapezoid = (area of parallelogram) ÷ 2 = $(b_1 + b_2)$ x h ÷ *2*

More often, the formula is written as:

$$\boxed{\mathbf{A = \frac{b_1 + b_2}{2} \text{ x } h}}$$

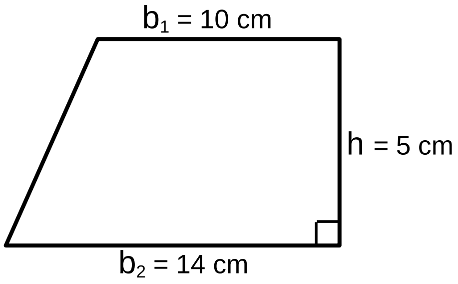
Now look at the examples involving the area of a trapezoid in the next page.

Here are the examples.

Example 1

Find the area of a trapezoid with bases 10 cm and 14 cm, and a height of 5 cm.

Solution: $b_1 = 10$ cm, $b_2 = 14$ cm, $h = 5$ cm



$$A = \frac{b_1 + b_2}{2} \times h$$

$$A = \frac{10 + 14}{2} \times 5$$

$$= \frac{24}{2} \times 5$$

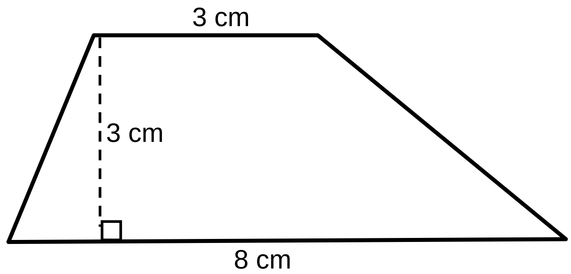
$$= 12 \times 5$$

$$= 60 \text{ cm}^2$$

Therefore the area of the trapezoid is **60 cm^2**.

Example 2

Calculate the area of the trapezoid shown below.



Solution: $b_1 = 3$ cm, $b_2 = 8$ cm, $h = 3$ cm

$$A = \frac{b_1 + b_2}{2} \times h$$

$$= \frac{3 \text{ cm} + 8 \text{ cm}}{2} \times 3 \text{ cm}$$

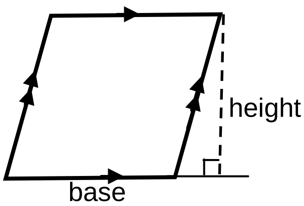
$$= \frac{11}{2} \times 3$$

$$= 16.5 \text{ cm}^2$$

Therefore, the area of the trapezium is **16.5 cm^2**.

Area of a Rhombus and a Kite

A **rhombus** is a special parallelogram with all sides equal, but with diagonals of different lengths. We can find the area of the rhombus in the same way as a parallelogram. That is:

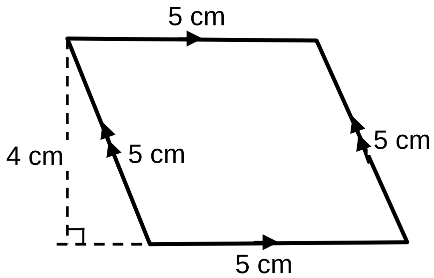


Area of a Rhombus = base x height

$$\boxed{\mathbf{A = b \times h}}$$

Example 1

Find the area of the rhombus at the right.



Solution:

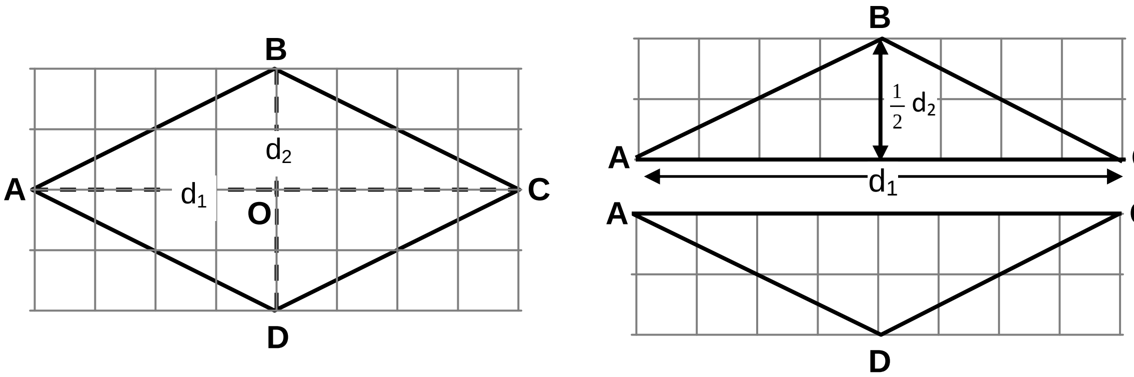
A = b x h

A = 5 cm x 4 cm

= 20 cm

We can also obtain the area of a rhombus, given the lengths of its diagonals. The diagonals of a rhombus bisect each other at right angles.

Look at the figures.



In this rhombus, the diagonal **AC** bisects diagonal **BD** at right angles at point **O**
If diagonal **AC** = $\mathbf{d_1}$ and diagonal **BD** = $\mathbf{d_2}$, then the base of triangle **ABC** = $\mathbf{d_1}$ and the perpendicular height = $\frac{1}{2}$ $\mathbf{d_2}$.

So, to find the area of the rhombus, we can either add the area of the two triangles or multiply the area of one triangle by 2.

Area of Rhombus **ABCD** = 2 x (Area of Δ **ABC**)

$= 2 \times \frac{1}{2}$(base x height)

Since the base of $\triangle$**ABC** is $\mathbf{d_1}$ and the height is $\frac{1}{2}\mathbf{d_2}$**,** then we have

$$\text{Area of Rhombus} = 2 \times \frac{1}{2} \times d_1 \times \frac{1}{2} d_2$$

$$= \frac{2}{4} d_1 d_2$$

$$= \mathbf{\frac{1}{2} d_1 d_2}$$

Therefore, using the diagonals:

Area of the rhombus = $\frac{1}{2}$ **the products of its diagonals.**

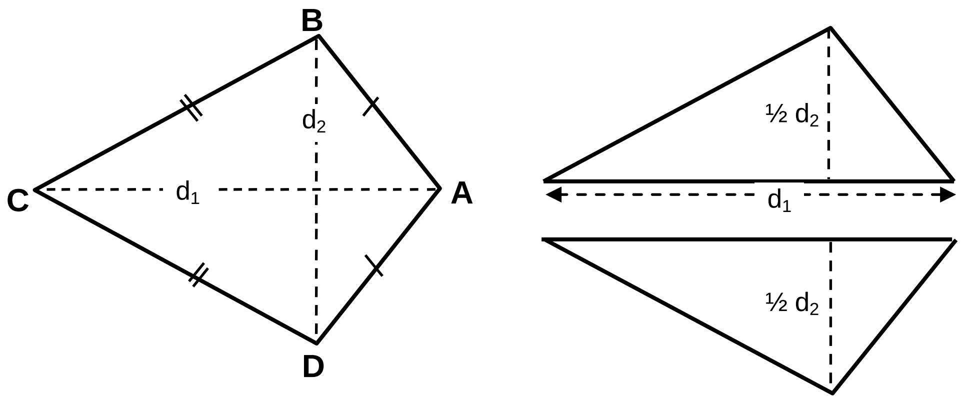
$$\boxed{\mathbf{A = \frac{1}{2} d_1 d_2}}$$

Remember $\mathbf{d_1}$ and $\mathbf{d_2}$ are the lengths of the two diagonals of the rhombus.

This formula can also be applied in finding the area of a kite.

As you have learnt, a **kite** is a quadrilateral with two pairs of adjacent sides equal.

Look at the figures.



Notice that the kite can be divided into two triangles.

So, to find the area of the kite, using the diagonals:

Area of the Kite = $\frac{1}{2}$ **the products of its diagonals.**

$$\boxed{\mathbf{A = \frac{1}{2} d_1 d_2}}$$

Now look at the examples.

Example 2

Find the area of the rhombus whose diagonals are 4 cm and 7 cm long as shown below.

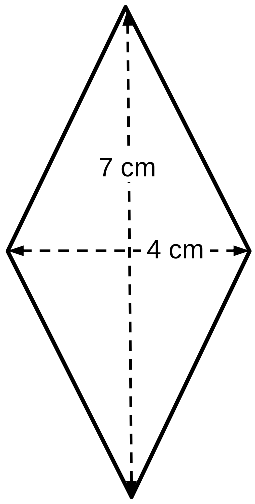
Solution: $d_1 = 7$ cm, $d_2 = 4$ cm

$$A = \frac{1}{2}d_1d_2$$

$$A = \frac{1}{2}(7\text{ cm})(4\text{ cm})$$

$$A = \frac{1}{2}(28\text{ cm}^2)$$

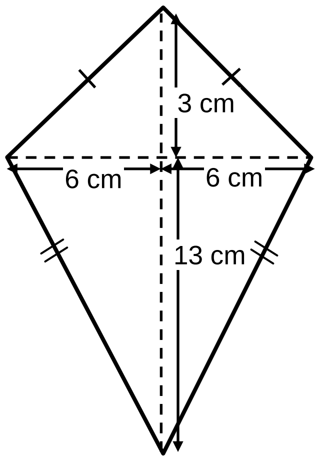
$$A = 14\text{ cm}^2$$



Therefore, the area of the trapezoid is **14 cm^2.**

Example 3

Find the area of the kite below.



Solution: $d_1 = 3\text{ cm} + 13\text{ cm} = 16\text{ cm}$
$d_2 = 6\text{ cm} + 6\text{ cm} = 12\text{ cm}$

$$A = \frac{1}{2}d_1d_2$$

$$A = \frac{1}{2}(16\text{ cm})(12\text{ cm})$$

$$= \frac{1}{2}(192\text{ cm}^2)$$

$$\mathbf{= 96\text{ cm}^2}$$

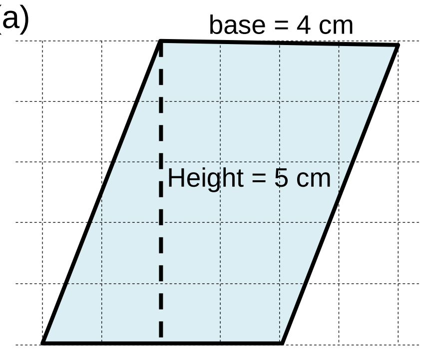
Therefore, the area of the trapezoid is **14 cm^2.**

NOW DO PRACTICE EXERCISE 12

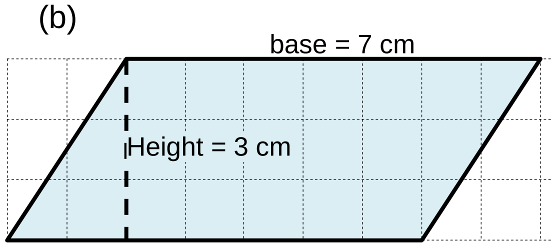
Practice Exercise 12

1. Find the area of the following parallelograms

(a)



(b)

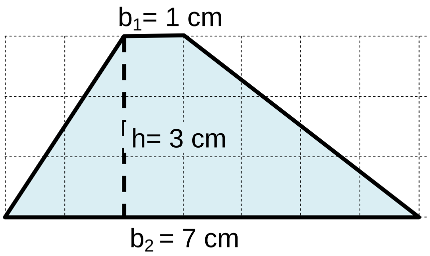


Answer: ______________

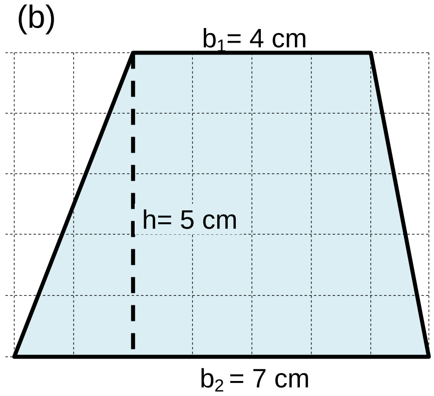
Answer: ______________

2. Find the area of the following trapezoids.

(a)



(b)

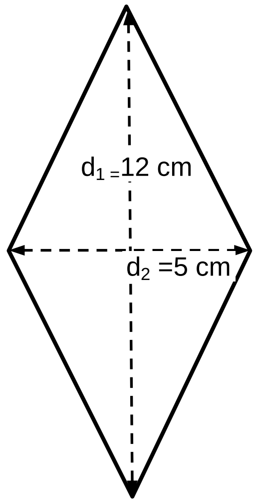


Answer: _______________

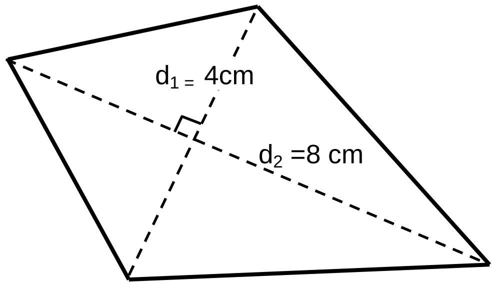
Answer: _______________

3. Find the area of the following rhombus and kite.

(a)



(b)

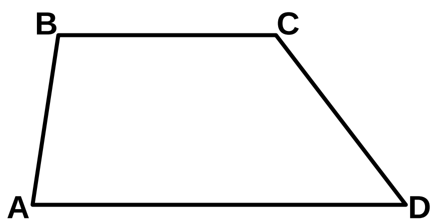


Answer: ______________ **Answer:** ______________

---

4. Solve the following problems.

(a) Find the area of trapezoid ABCD given BC = 9.7 cm, AD = 24.3 cm, and a height of 8.5 cm.

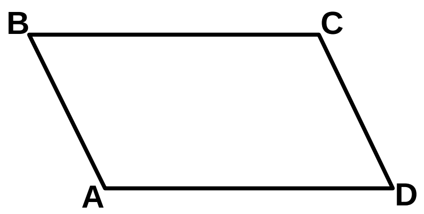


Answer:____________

(b) A rhombus has a side length of 6 units and an altitude of 5 units. What is the area of the rhombus?

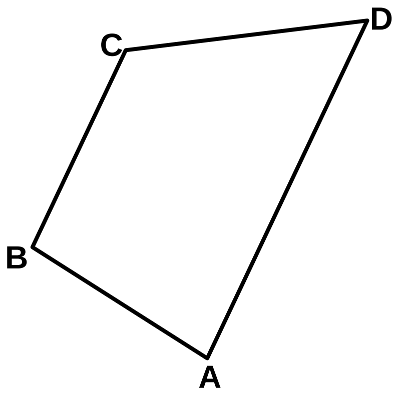
Answer:____________

(c) AD measures 8 units in parallelogram ABCD. If the area of ABCD is 44 square units, what is the measure of the corresponding altitude?



Answer:___________

(d) Trapezoid ABCD has an area of 57 square units. BC = 7 units and AD = 12 units. What is the height of the trapezoid?

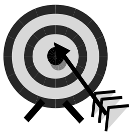


Answer:___________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

Lesson 13: Area of Compound Shapes

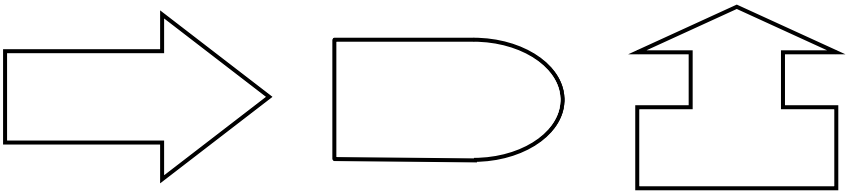
In Grade 8, we learnt how to find the area of composite shapes by dividing them into rectangles and triangles.



In this lesson, you will:

- revise the meaning of compound shapes and find the area of more compound or composite shapes.

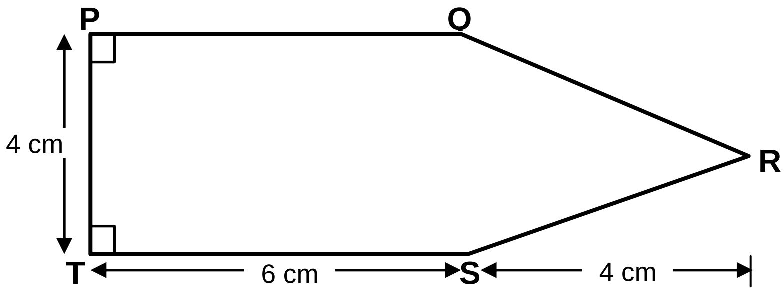
Look at the shapes below.



These are all examples of compound or composite shapes.

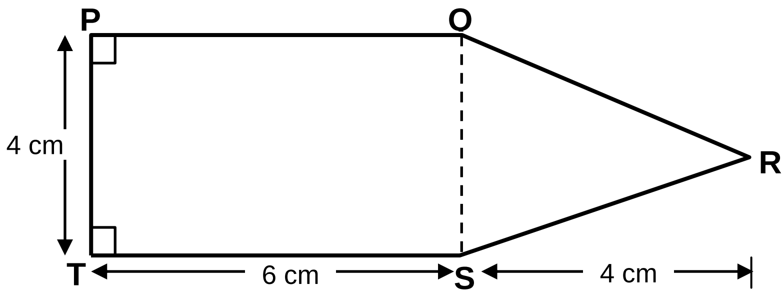
First, let us recall what a compound or composite shape is.

Look at the shape below.



If we are to find the area of this shape, none of the formulae we have learnt can be used to calculate the area of this shape as it is. To find its area, we must first **divide** it into simple shapes.

We do this by drawing a dotted line between **Q** and **S**, thus forming two different shapes.



Notice that a rectangle and a triangle are formed.

So, now we can define a composite or compound shape.

> A **composite or compound shape** is a shape that can be divided into more than one of the basic figures such as triangles, rectangles, squares and even circles or semi-circles.

Now, we calculate the area of the rectangle and triangle separately.

Area of rectangle = L **x** W ....L= 6 cm, W = 4 cm
= 6 cm **x** 4 cm
= **24 cm^2**

Area of triangle = $\frac{1}{2}$ **x** b **x** h .... b = 4 cm, h = 4 cm
= $\frac{1}{2}$ **x** 4 **x** 4
= **8 cm^2**

Add the two areas to get the total area of the compound or composite shape.

Total area = Area of rectangle + Area of triangle.
= 24 + 8
= **32 cm^2**

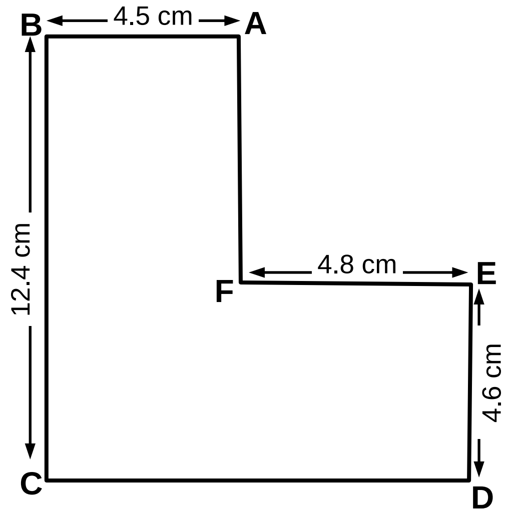
Therefore, the area of the composite shape is **32 cm^2**.

Sometimes you can find the area by adding areas, at other times it is easier to subtract.

Here is another example of compound shape.

Example 2

Find the area of the shape **ABCDEF**.



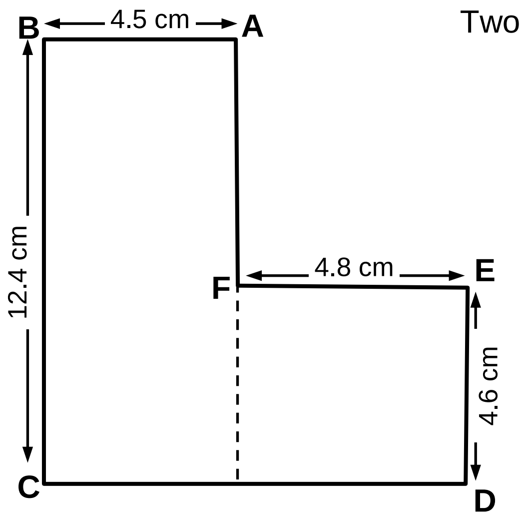
The area of **ABCDE** can be given by two methods:

1 By adding areas

2 By subtracting areas

Method 1 By adding areas

This can be done by drawing a horizontal line as shown below.



Two rectangles are formed. Rectangles **X** and **Y**.

:

Area of **ABCDEF** = area of **X** + Area of **Y**

Calculate the area of the rectangles separately.

Area of **X** = L x W ----------------------L = 12.4 cm, W = 4.5 cm

= 12.4 cm x 4.5 cm

= 55.8 cm^2

Area of **Y** = L x W-------------------------L = 4.8 cm, W = 4.6 cm

= 4.8 cm x 4.6 cm

= 22.08 cm^2

Add the two areas to get the area of shape **ABCDE**.

Area of **ABCDEF** = area of **X** + Area of **Y**

A = 55.8 cm^2 + 22.08 cm^2

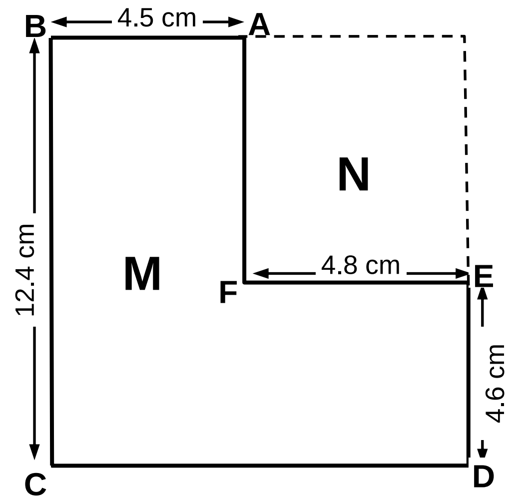
A = 77.9 cm^2 (rounded off to 1 dec. place)

Therefore, the area of **ABCDEF** is **77.9 cm^2.**

Now let us look at the second method.

Method 2 By subtracting areas

This method can be done by first completing the rectangle by drawing a dashed line along and down the top right hand corner as shown on the next page.



Now we have 2 complete rectangles which is not part of the original problem.

We will call the big rectangle **M** and the small rectangle **N.**

The area of **ABCDEF** can be given by:

Area of **ABCDEF** = area of **M** – Area of **N**

Calculate the area of **M** and area of **N** separately.

Area of **M** = L x W -----------------**L** = 12.4 cm, **W** = 4.5 cm + 4.8 cm = 9.3 cm

= 12.4 cm x 9.3 cm

= 115.32 cm^2

Area of **N** = L x W ----------------**L** = 12..4 cm – 4.6 cm = 7.8 cm, **W** = 4.8 cm

= 7.8 cm x 4.8 cm

= 37.44 cm^2

Subtract the two areas to find area of **ABCDE**.

Area of **ABCDEF** = area of **M** – Area of **N**

= 115.32 cm^2 – 37.44 cm^2

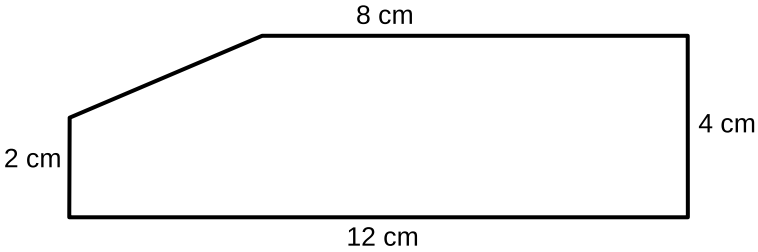
= **77.9 cm^2** (rounded off to 1 dec. place)

Therefore, the area of shape **ABCDEF** is **77.9 cm^2.**

Now look at the other examples on the next page.

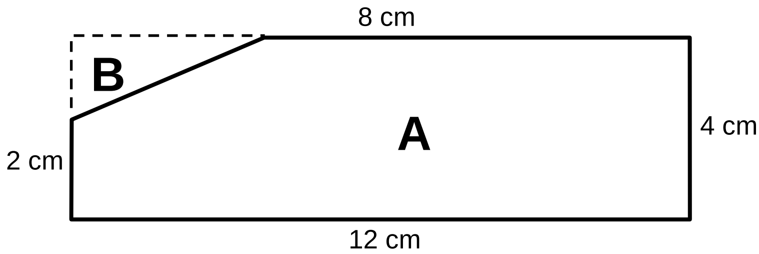
Example 3

Find the area of this shape.



First, complete the rectangle by drawing a dotted line along and down the top left hand corner.

Now we have a complete **rectangle** and a **triangle** which are not part of the original problem.



Calculate the area of the rectangle and the triangle separately. We will call the whole rectangle, **A** and the triangle, **B**.

Area of **A** = L x W ----------------- **L =** 12 cm, **W =** 5 cm

= 12 cm x 4 cm

= **48 cm$^{\mathbf{2}}$**

We will make the base of the triangle, the horizontal dotted line and the height, the vertical dotted line.

To find the base of the triangle, we subtract 8 cm from 12 cm. For the height, we subtract 2 cm from 4 cm.

Area of **F** = $\frac{1}{2}$ **x** b **x** h ------------------- **b =** 4 cm **,** **h =** 2 cm

= $\frac{1}{2}$ **x** 4 **x** 2

= **4 cm$^{\mathbf{2}}$**

Subtract the area of **B** from the area of **A** to obtain the area of the original shape.

Area of shape = Area of **A** – Area of **B**

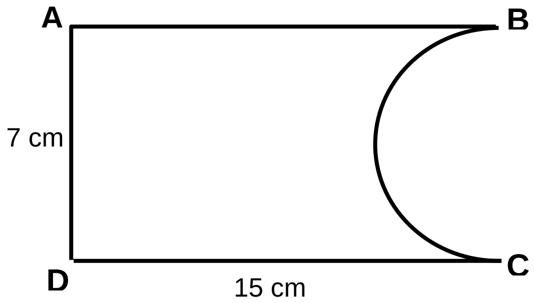
= 48 cm^2 – 4 cm^2

= **44 cm$^{\mathbf{2}}$**

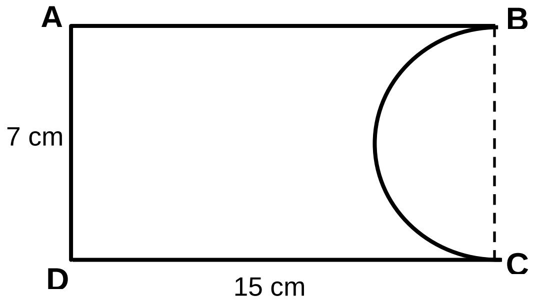
Therefore, area of the original shape is **44 cm$^{\mathbf{2}}$**.

Example 4

What is the area of this shape?



First, complete the rectangle by drawing a dotted line from **B** down to **C** as shown on the diagram.



Now we have a complete **rectangle** and a **semi-circle** which are not part of the original problem.

Calculate the area of the rectangle and the semi-circle separately. We will call the whole rectangle, **R** and the semi-circle, **S**.

Area of **R** = L x W ----------------- **L =** 15 cm, **W =** 7 cm
= 15 cm x 7 cm
= **105 cm^2**

We will make the diameter of the semi-circle, the dotted line.

To find the radius of the semi-circle , we divide 7 cm by 2.

Area of **S** = $\frac{1}{2}\pi r^2$ -------------------- **r =** $\frac{7}{2}$ **= 3.5 cm**

= $\frac{1}{2}$ x 3.14 x (3.5 cm)^2

= $\frac{1}{2}$ x 38.47 cm^2 --------→ (3.14 x 12.25 = 38.465)

= 19.2 cm^2 (rounded off to 1 dec. place)

Subtract the area of **S** from the area of **R** to obtain the area of the original shape.

Area of shape = Area of **R** – Area of **S**
= 105 cm^2 – 19.2 cm^2
= **85.8 cm^2**

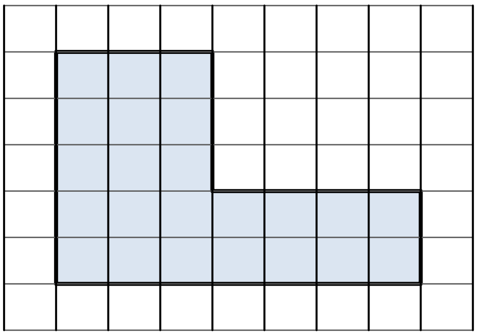
Therefore, area of the original shape is **85.8 cm^2**.

NOW DO PRACTICE EXERCISE 13

Practice Exercise 13

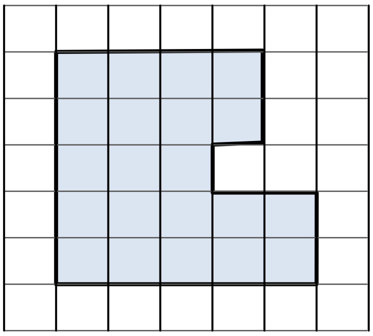
1. Find the area of the following compound or composite shapes (shaded squares in the grid). One square on the grid is 1 cm^2.

(a)



Answer: ___________

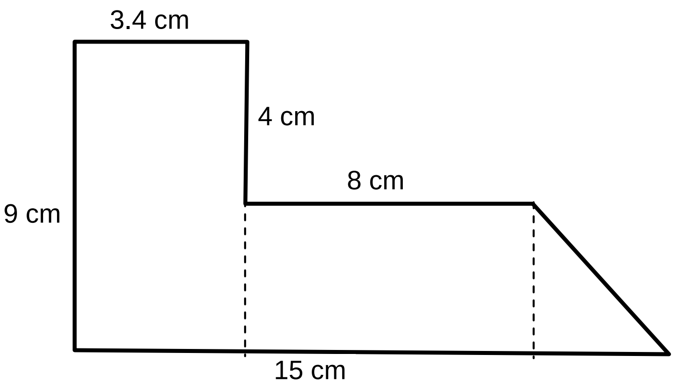
(b)



Answer: ___________

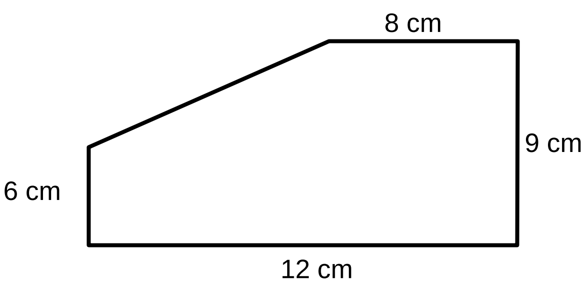
2. Find the area of the following shapes.

(a)



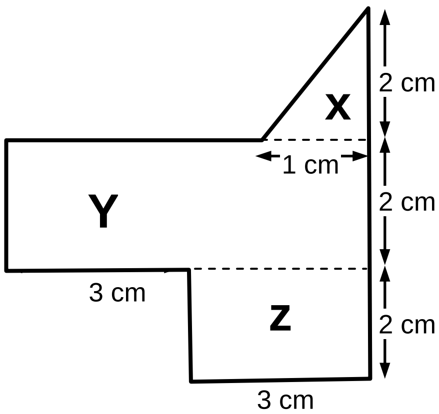
Answer: ____________

(b)



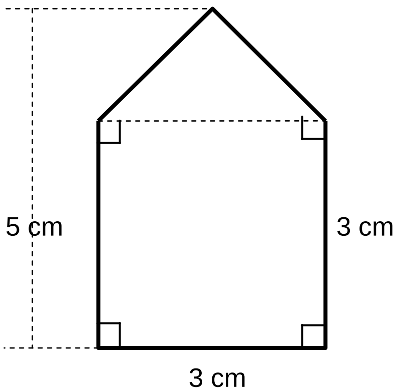
Answer: ____________

(c)



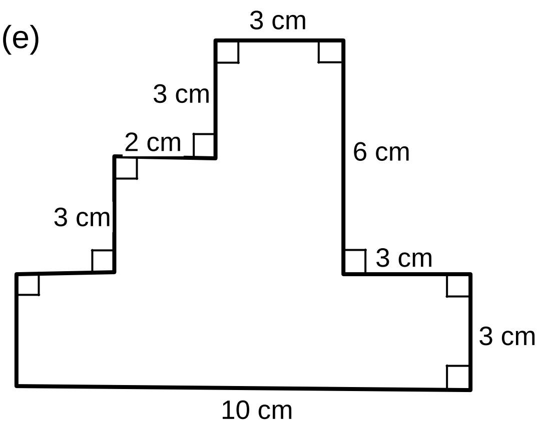
Answer: ____________

(d)



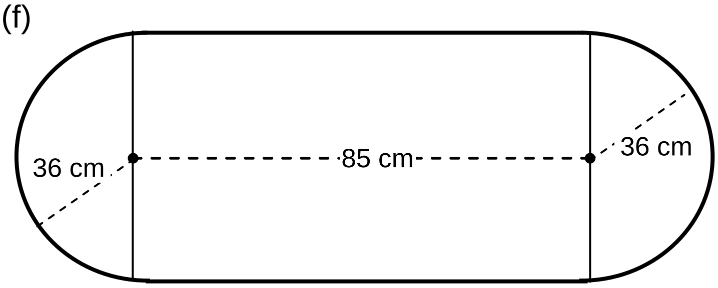
Answer: ____________

(e)



Answer: ____________

(f)

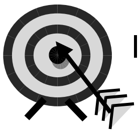


Answer: ____________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

Lesson 14: Applied Problems on Area

You have learnt about the different formula used to calculate area of different shapes in the previous lessons.



In this lesson, you will:

- solve problems involving area in real life situations.

Learning to calculate area is an important skill we need to understand in order for us to be able to solve real world problems such as figuring out how much carpet we need for a room, how much dough you need for a pizza, and how much floor space you need in a warehouse.

In this lesson you will find examples with solutions of the common shapes in use in the real world and you may see them in everyday life.

Example 1

Joyce has a rectangular rose garden that measures 6 m by 15 m. One bag of fertilizer can cover 10 m^2. How many bags will she need to cover the entire garden?

Solution:

Step 1: Find the area of the rose garden first and then calculate how many bags are needed.

Since the garden is rectangular, use the formula: **A = L x W**

L = 15 m, W = 6 m

Substituting these in the formula, we have

$$A = L \times W$$
$$A = 15 \text{ m} \times 6 \text{ m}$$
$$= 90 \text{ m}^2$$

Step 2: A 10 m^2 area is covered using one bag of fertilizer.

To get the number of bags needed, divide the area by 10 m^2.

$$\text{Number of bags} = \frac{90\text{ m}^2}{10\text{ m}^2}$$

$$= 9$$

Therefore, to cover 90 m^2 area will need 9 bags of fertilizer.

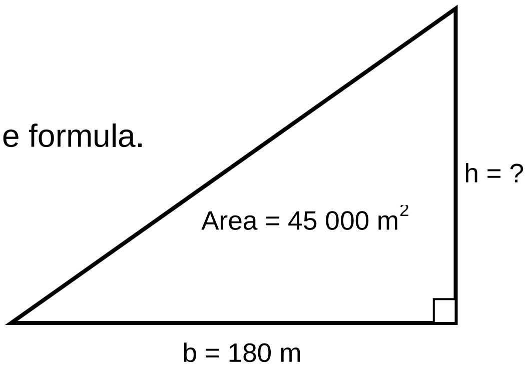
Example 2

Richard is surveying a plot of land in the shape of a right triangle. The area of the land is 45,000 square meters. If one leg of the triangular plot is 180 meters long, what is the length of the other leg of the triangle?

Solution:

This is a problem involving area of triangle. Since it's a right triangle, one leg can be considered as the base, and the other as the height.

Use the formula: $A = \frac{1}{2}bh$



Substitute 45000 m^2 for **A** and 180 m for **b** in the formula.

$$45\ 000\ m^2 = \frac{1}{2}(180\ m) \times h$$

Simplify $45\ 000\ m^2 = 90\ m \times h$

Divide both sides by 90 m.

$$\frac{45\,000}{90} = \frac{90h}{90}$$

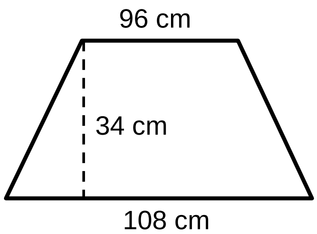
500 = h or h = 500 m

Therefore, the other leg of the triangular plot of land is 500 m long.

Example 3

The counter at Molly's Diner is shaped like a trapezoid. Molly plans to have a new surface applied to the counter which will cost K0.06/cm^2 to purchase and apply.

At the right is a diagram of the counter which the carpenter made in order to plan the job. How much will Molly have to pay to have the job done?



Solution: Use the formula: $A = \frac{b_1 + b_2}{2} \times h$

b_1 = 96 cm, b_2 = 108 cm, h = 34 cm

Substitute these in the formula, we have

$$A = \frac{96\ cm + 108\ cm}{2} \times 34\ cm$$
$$= \frac{204\ cm}{2} \times 34\ cm$$
$$= 102\ cm \times 34\ cm$$
$$= 3468\ cm^2$$

To find the cost of the job, multiply the area by the cost per cm^2.

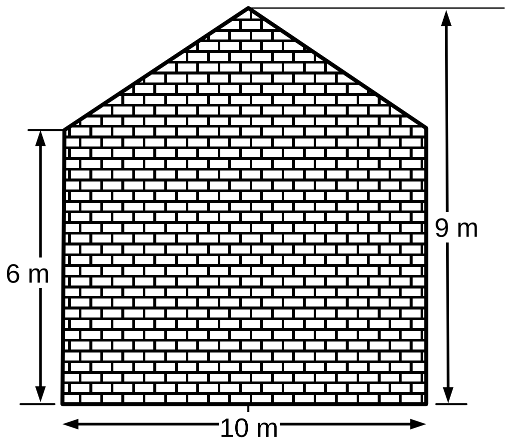
Cost = K0.06 x 3468

= K208.08

Hence, Molly will have to pay K208.08 to the carpenter.

Example 4

A wall is in the shape of a triangle on top of a rectangle with the measurements shown on the diagram below. Find the area in m^2 of the wall?



Solution: This problem involves an area of compound shape. The wall is made up of a triangle and a rectangle.

To find the area of the wall we need to find the area of the triangle and the area of the rectangle then add them.

So Area of the wall = area of rectangle + area of triangle.

Calculate the two areas separately.

Area of rectangle = L x W --------L = 10 m, W = 6 m

$$A = 10 \text{ m} \times 6 \text{ m}$$
$$= 60 \text{ m}^2$$

Area of triangle = $\frac{1}{2}bh$ -----------b = 10 m, h = 9m – 6m = 3 m

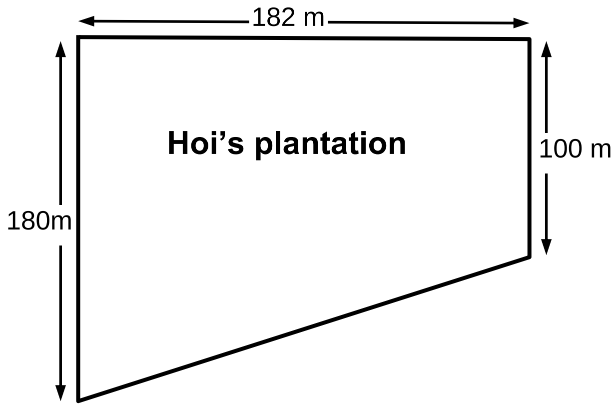
$$= \frac{1}{2}(10 \text{ m})(3 \text{ m})$$
$$= \frac{1}{2}(30 \text{ m}^2)$$
$$= 15 \text{ m}^2$$

Hence, the Area of the wall = $60 \text{ m}^2 + 15 \text{ m}^2$

$$= 75 \text{ m}^2$$

Therefore the area of the wall is 75 $\mathbf{m^2}$.

4. The figure below shows the area covered by Hoi's cocoa plantation.



(a) Find the area of the plantation in square metres (m^2).

(b) Find the area in hectares.

Solution:

(a) The plantation is shaped-like a trapezoid. Use the formula of the area of trapezoid.

$$A = \frac{b_1 + b_2}{2} \text{ x } h$$

b_1 = 100 m, b_2 = 180 m, h = 182 m

Substitute these in the formula, we have

$$A = \frac{100\,\text{m} + 180\,\text{m}}{2} \text{ x 182 cm}$$

$$= \frac{280\,\text{m}}{2} \text{x 182 cm}$$

= 140 m x 182 cm

= 25480 m^2

(b) Area in Hectares = $\frac{25480}{10\,000}$ (1 ha = 10 000 m^2)

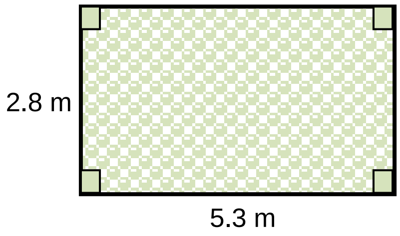
= 2.548 ha

NOW DO PRACTICE EXERCISE 14

Practice Exercise 14

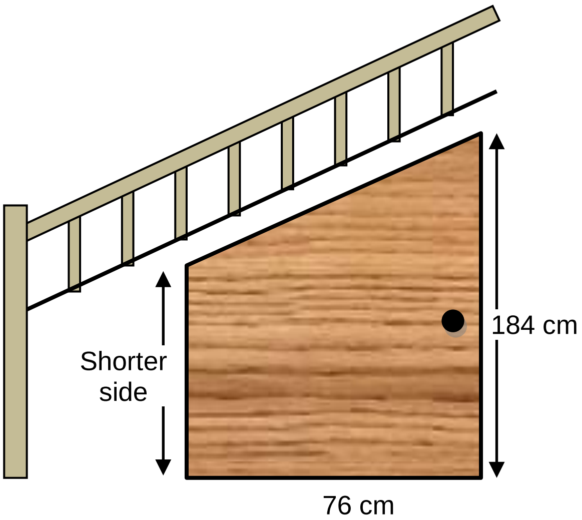
Solve the following word problems.

1. A wall is 5.3 m long and 2.8 m high. Find the cost of painting this wall at K5.50 per m^2.



Answer:___________

2 The diagram shows the door of a cupboard under some stairs. One side of the door is shorter than the other by 32 cm. The door is 76 cm wide.



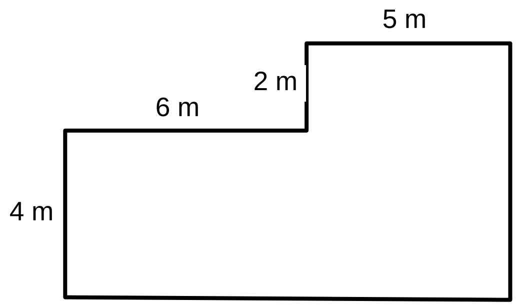
(a) What is the shape of the door?

Answer: ___________

(b) Calculate the area of the front door.

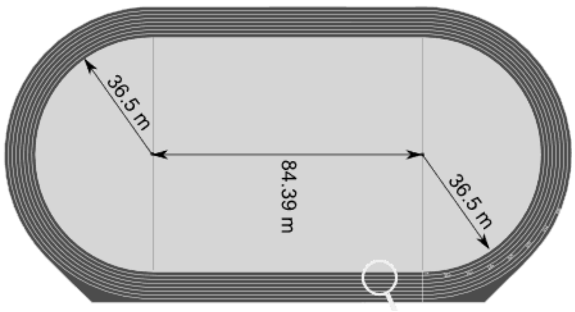
Answer: ___________

3. How much fertilizer would be needed to cover a lawn with the measurements shown below if 1 kg will cover 10 m^2?



Answer: __________

4. The figure below shows the area bounded by a standard race track.



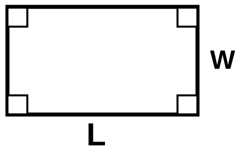
(a) Find the area bounded by the track in square metres.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

TOPIC 3: SUMMARY

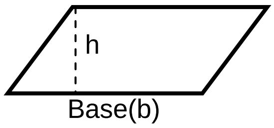
This summarizes some of the important ideas and concepts to be remembered.

- The area of a rectangle is given by:



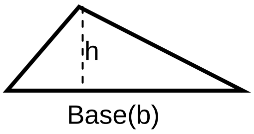
A = Length x Width or A = L x W

- The area of a parallelogram is given by:



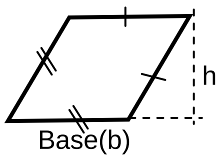
A = base x height or A = bh

- The area of a triangle is given by:



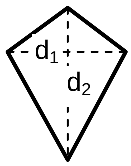
$$\mathbf{A = \frac{1}{2}(Base \times height) \text{ or } A = \frac{1}{2}bh}$$

- The area of a rhombus is given by:



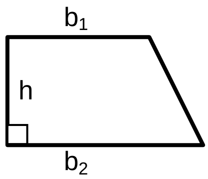
A = base x height or A = bh

- The area of a kite is given by;



$$\mathbf{A = \frac{1}{2}d_1d_2}$$

- The area of trapezoid is given by:



$$\mathbf{A = \frac{b_1 \times b_2}{2} \times h}$$

- **A compound or composite shape** is a shape that can be divided into more basic shapes such as rectangles, squares, or triangles.
- The area of a composite or compound shape can be found **by adding together or subtracting areas of the common shapes.**

REVISE LESSONS 11- 14. THEN DO TOPIC TEST 3 IN ASSIGNMENT BOOK 5.

ANSWERS TO PRACTICE EXERCISES 11-14

Practice Exercise 11

1. (a) 448 m^2 (b) 324 m^2 (c) 45.5 cm^2 (d) 35.4 m^2
2. (a) 54 cm^2 (b) 76 cm^2
3. (a) 50 cm (b) 5 cm

Practice Exercise 12

1. (a) 20 cm^2 (b) 21 cm^2
2. (a) 12 cm^2 (b) 47.5 cm^2
3. (a) 30 cm^2 (b) 16 cm^2
4. (a) 144.5 cm^2 (b) 30 sq. units (c) 5.5 units (d) 6 units

Practice Exercise 13

1. (a) 23 cm^2 (b) 21 cm^2
2. (a) 79.6 cm^2 (b) 102 cm^2 (c) 14 cm^2 (d) 12 cm^2
   (e) 54 cm^2 (f) 10 189.44 cm^2

Practice Exercise 14

1. K81.62
2. (a) trapezoid (b) 12 768 cm^2
3. 5.4 kg
4. 10343.735 m^2

END OF TOPIC 3

TOPIC 4

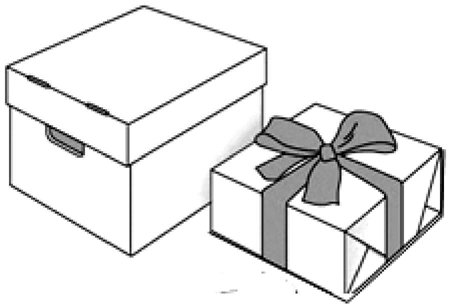
SURFACE AREA AND VOLUME

TOPIC 4: SURFACE AREA AND VOLUME

Introduction

Have you ever wrapped a birthday gift? If so, then you've covered the surface area of a polyhedron with wrapping paper.

Have you ever poured yourself a glass of milk? If so, then you have filled the volume of a glass with liquid.



Surface area is exactly what it sounds like — the area of all of the outside surfaces of a three-dimensional object. And **volume** is all of the space inside a three-dimensional object.

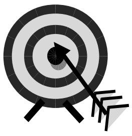
In this lesson, you will learn more about both of these concepts as well as how to compute the surface area and volume of various space figures like polyhedron, cones, cylinders and spheres.

Polyhedrons are space figures with flat surfaces called faces which are made of polygons. Prisms and pyramids are examples of polyhedrons.

Cylinders, cones and spheres are not polyhedrons, because they have curved not flat surfaces.

Lesson 15: Surface Area of Prisms

You learnt the meaning of surface area in your Grade 8 Mathematics. You also learnt the different formulas in finding the surface area of different types of prisms.



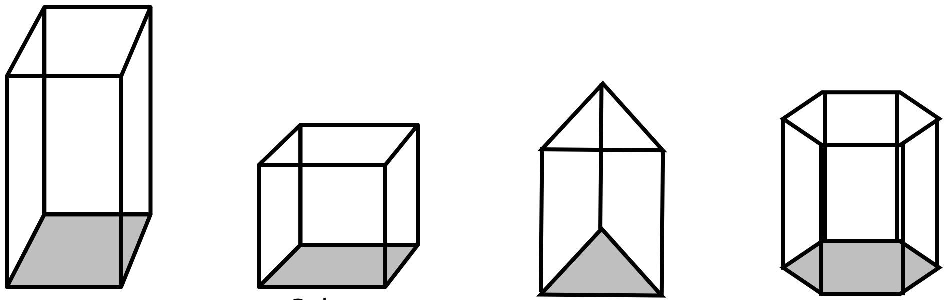
In this lesson, you will:

- revise prisms
- define surface area
- calculate the surface area of prisms

First, let us revise the meaning of a prism.

A **prism** is a polyhedron, with two parallel faces called **bases**. The other faces are always **parallelograms**. The prism is named by the shape of its base.

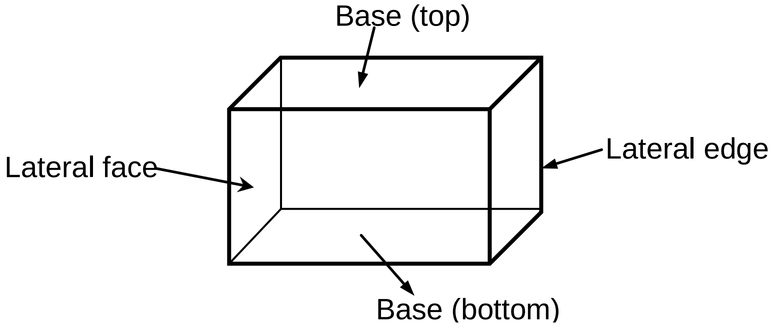
Here are some types of prisms. They come in many shapes and sizes.



Rectangular Prism   Cube   Triangular Prism   Hexagonal Prism

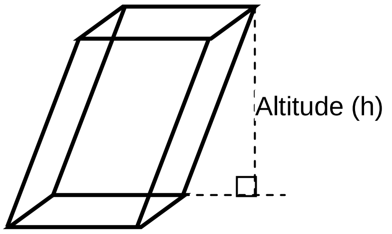
Every prism has the following characteristics:

- **Bases:** A prism has two bases, which are congruent polygons lying in parallel planes.
- **Lateral edges:** The lines formed by connecting the corresponding vertices, which form a sequence of parallel segments.
- **Lateral faces:** The parallelograms formed by the lateral edges.

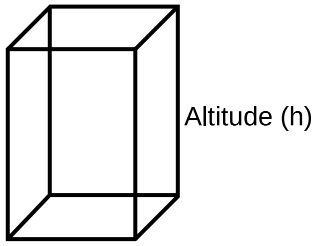


A prism is named by the polygon that forms its base, as follows:

- **Altitude:** A segment perpendicular to the planes of the bases with an endpoint in each plane.
- **Oblique prism:** A prism whose lateral edges are not perpendicular to the base.
- **Right prism:** A prism whose lateral edges are perpendicular to the bases. In a right prism, a lateral edge is also an altitude.



Oblique Prism



Right Prism

Now let us look at the surface area of a prism. First let us define surface area.

We learnt that the area of a shape is nothing more than the sum of all the unit squares of a shape. For the surface area, there is a similar definition, but it applies to the lateral surfaces of the solid.

The surface area of any solid is equal to the sum of the area of all of its lateral faces.

Surface Area of General Prisms

The sum of all the faces of a rectangular prism is called the **total surface area (TSA).** The sum of the areas of the four faces that are not bases is called the **lateral surface area (LSA)**.

The general formula for the lateral surface area and the total surface area of prisms are given by the following formulas:

For the lateral surface area

$$\boxed{\mathbf{LSA = p \times H}}$$

Where: p = perimeter of the base
H = height of the prism

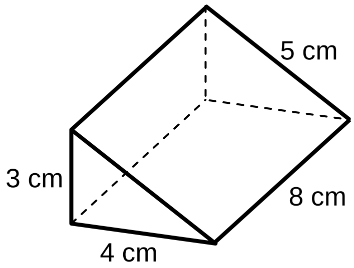
For the total surface area

$$\boxed{\mathbf{TSA = LSA + 2B}}$$

Where: LSA = lateral surface area
B = area of the base of the prism

Example 1

A triangular prism has bases whose edges are 3 cm, 4 cm and 5 cm and a height of 8 cm. Find: (a) Lateral area
(b) Total surface area



Solution:

(a) Lateral area is given by: LSA = p x H

The perimeter is the sum of sides of the base. p = 3 + 4 + 5 = 12 cm
H = 8 cm

Substituting these in the formula we have

$$\begin{aligned} LSA &= p \times H \\ &= 12 \times 8 \\ &= \mathbf{96\ cm^2} \end{aligned}$$

Therefore the lateral surface area of the triangular prism is 96 cm^2.

(b) Total Surface area Is given by: TSA = LSA + 2B

Since the prism is triangular, its bases are triangles. To find the area of the base we use the formula:

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(3)(4)$$

$$= \mathbf{6\ cm^2}$$

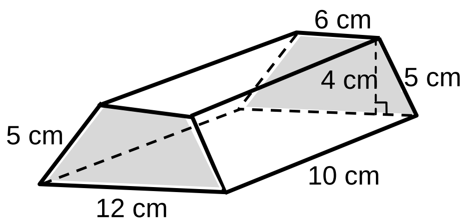
Now we can compute the total surface area knowing LSA = 96 cm^2 and B = 6 cm^2. Substitute these in the formula, we have:

$$\begin{aligned} TSA &= LSA + 2B \\ &= 96\ cm^2 + 2(6cm^2) \\ &= 96\ cm^2 + 12\ cm^2 \\ &= \mathbf{108\ cm^2} \end{aligned}$$

Therefore the total surface area of the triangular prism is 108 cm^2.

Example 2

Find the total surface area of an isosceles trapezoidal prism with parallel edges of the base 6 cm and 12 cm, the legs of the base are 5 cm each, the altitude of the base is 4 cm and the height of the prism is 10 cm.



Solution:

The total surface area is given by the formula: TSA = LSA + 2B, where LSA is the lateral area and B is the area of the base.

First we find the lateral surface area using the formula: LSA = p x H,

The perimeter of the trapezoidal base: p = 5 cm + 6 cm + 5 cm + 12 cm = **28 cm**

H = 4 cm

Hence, the lateral surface area is: LSA = 28 cm x 10 cm

= 280 cm^2

Now we find the area of the trapezoidal base (B). Use the formula $A = \frac{b_1 + b_2}{2} \times h$

$b_1 = 6$ cm; $b_2 = 12$ cm; $h = 4$ cm

Substitute these in the formula, we have:

$$A = \frac{6\text{ cm} + 12\text{ cm}}{2} \times 4\text{ cm}$$

$$= \frac{18\text{ cm}}{2} \times 4\text{ cm}$$

$$= 9\text{ cm} \times 4\text{ cm}$$

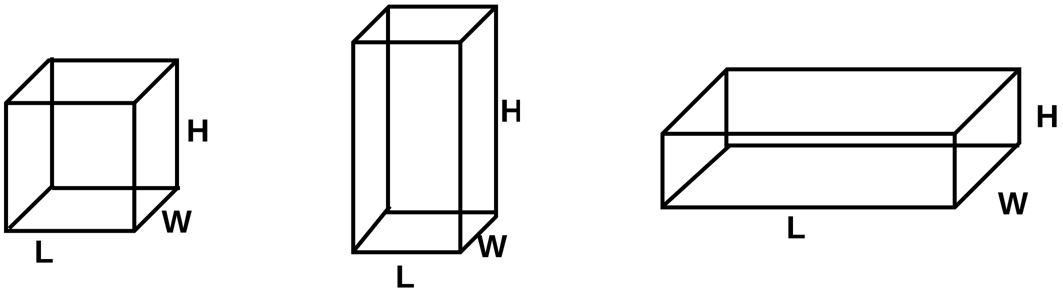
$$\mathbf{= 36\text{ cm}^2}$$

Now we can compute the total surface area knowing LSA = 280 cm^2 and B = 36 cm^2. Substitute these in the formula, we have:

$$\begin{aligned} TSA &= LSA + 2B \\ &= 280\text{ cm}^2 + 2(36\text{cm}^2) \\ &= 280\text{ cm}^2 + 72\text{ cm}^2 \\ &= 352\text{ cm}^2 \end{aligned}$$

Therefore the total surface area of the trapezoidal prism is 352 cm^2.

Now look at the figures below.



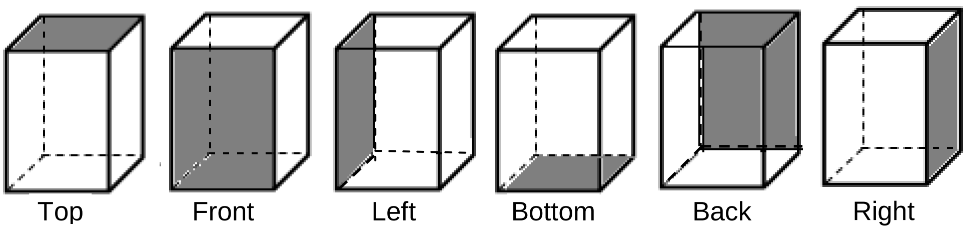
We called these box-shaped objects **cuboids**. They all have six flat sides and all angles are right angles. They are also called prisms because they have the same cross section along a length. All the surfaces of the prisms are rectangular. In fact they are called **rectangular prisms**. In a cuboid or rectangular prism, the length, width and height may be of different lengths. When at least two of the dimensions are equal the cuboid can also be called **square prism**. If all the three dimensions are equal it can be called a **cube.**

Remember:

- a cube is just a special case of a square prism, and
- a square prism is just a special case of a rectangular prism, and
- they are all cuboids!

As the diagrams indicate, there are six lateral surfaces to each rectangular prism. There is the front, back, top, bottom, left and right to every rectangular prism.

SURFACE OF A RECTANGULAR PRISM OR CUBOID



The sum of all the faces of a rectangular prism is called the **total surface area (TSA).** The sum of the areas of the four faces that are not bases is called the **lateral surface area (LSA)**.

Using the labelling of the prisms on the diagram above, a formula can be created for dealing with the surface area of cuboids or rectangular prisms. Let us calculate the area of each surface.

PRISM SURFACE AREA FORMULA

Top	:	A = LW
Bottom	:	A = LW
Front	:	A = HL
Back	:	A = HL
Left	:	A = HW
Right	:	A = HW

To calculate the surface area of the cuboid or rectangular prism, we need to calculate the area of each surface and then add all the areas to get the total surface area.

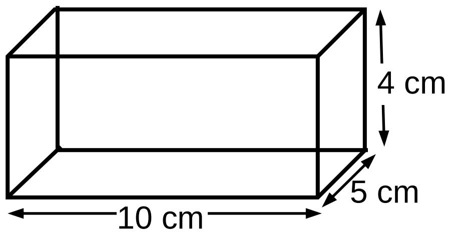
Total area of top and bottom surfaces = 2(LW)
Total area of front and back surfaces = 2(HL)
Total area of left and right surfaces = 2(HW)

So, the Total Surface Area = 2(LW) + 2(HL) + 2(HW) or

TSA = 2(LW + HL + HW)

Example 1

Find the total surface area of this cuboid.



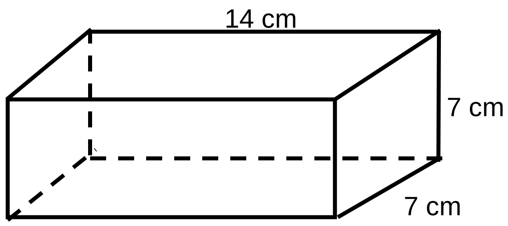
Solution:

$$\begin{aligned} TSA &= 2(LW + HL + HW) \\ &= 2[(10\text{ cm})(5\text{ cm}) + (4\text{ cm})(10\text{ cm}) + (4\text{ cm})(5\text{ cm})] \\ &= 2(50\text{ cm}^2 + 40\text{ cm}^2 + 20\text{ cm}^2) \\ &= 100\text{ cm}^2 + 80\text{ cm}^2 + 40\text{ cm}^2 \\ &= 220\text{ cm}^2 \end{aligned}$$

Therefore, the total surface area of the cuboid is 220 cm^2.

Example 2

A rectangular tank is 14m long, 7 m wide and 7 m high. Calculate the total surface area of the tank.



Solution: TSA = TSA = 2(LW + HL + HW)

Total area of top and bottom surfaces = 2(14)(7) = 196 cm^2
Total area of front and back surfaces = 2(14)(7) = 196 cm^2
Total area of left and right surfaces = 2(7)(7) = 98 cm^2

TSA of rectangular prism = 196 cm^2 + 196 cm^2 + 98 cm^2
= 490 cm^2

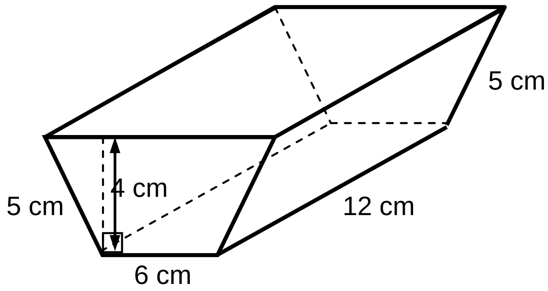
Therefore, the total surface area of the prism is 490 cm^2.

NOW DO PRACTICE EXERCISE 15

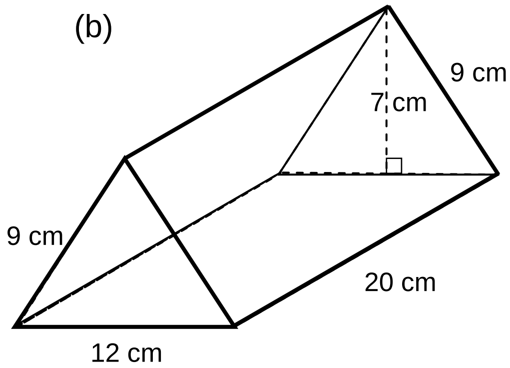
Practice Exercise 15

1. Calculate the surface area of each prism below.

(a)



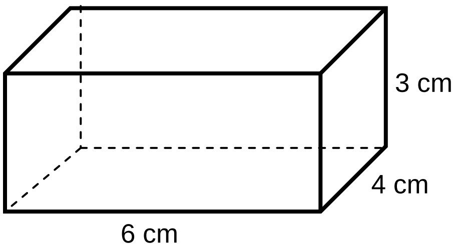
(b)



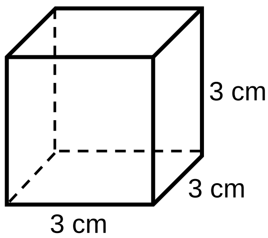
Answer:___________

Answer:___________

(c)



(d)

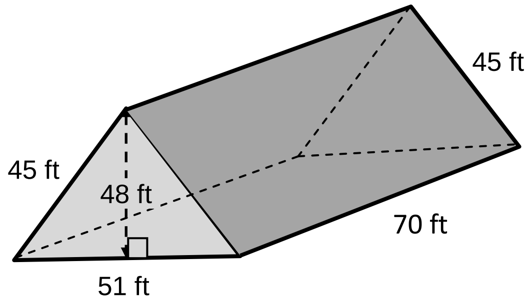


Answer:___________

Answer:___________

2. Problem Solving

(a) Find the surface area of a triangular tent whose dimensions are shown below.

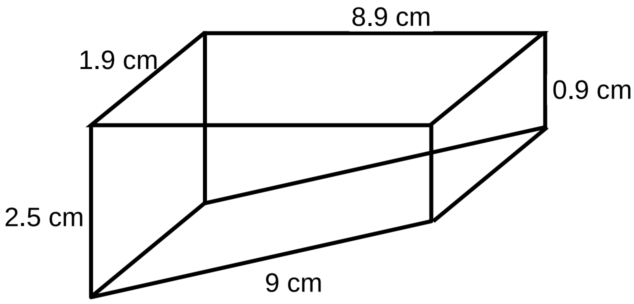


Answer: __________

(b) Find the surface area of a rectangular prism with a length of 24 cm, a width of 20 cm, and a height of 15 cm.

Answer: __________

(c) Find the surface area of a trapezoidal prism below given the dimensions as shown.



Answer: __________

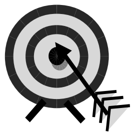
(d) A cubical tank with sides 8.9 m is to be painted. It costs K2 per m^2 to paint. Find the total cost to paint all the six sides.

Answer: __________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 16: Volume of Prisms

You learnt about different formulas in finding the surface area of different types of prisms in the previous lesson.



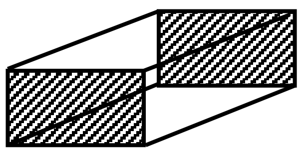
In this lesson, you will:

- revise your existing knowledge on prisms
- calculate the volume of the different types of prisms such as cubes and cuboids, triangular prism and trapezoidal prisms.

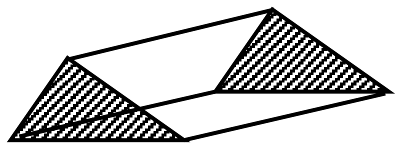
First let us revise some of the work we did in Grade 8 on prisms.

As we have learnt, prisms have two important features:

(i) They have **two parallel identical end faces** (shaded).

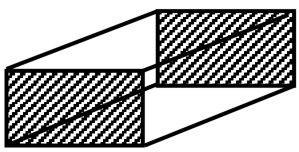


The end face is a rectangle. So, it is called a rectangular prism or cuboid

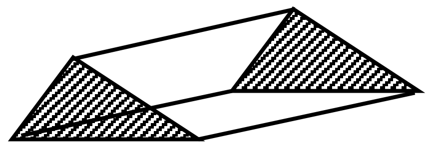


The end face is a triangle. So, it is called a triangular prism.

(ii) Prisms have **side faces** in the shape of parallelograms that are normally **rectangular**.



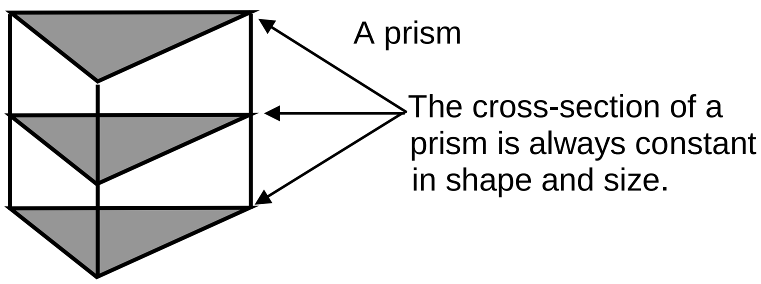
4 rectangular side face



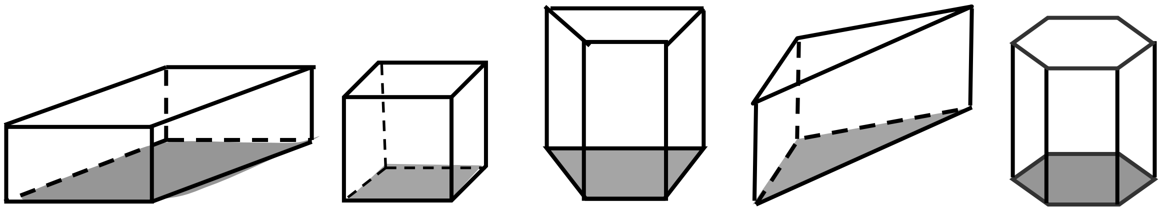
3 rectangular side face

Cross-section of a prism

The cross-section of a prism is always the same in shape and size, unlike for example, in a pyramid. That is, a prism must have a uniform or constant cross-section.



Look at the following figures. These are all prisms. Notice that in each case, the base is used as the **cross-section**.



Rectangular prism Cube Trapezoidal prism Triangular prism Hexagonal prism

Volume of a Prism

To calculate the volume of any prism, we have to know:

(i) the area of its cross-section or base

(ii) the height

Volume of a prism = Area of cross-section or Base x height

$$\boxed{\mathbf{V = B \times h}}$$

Example 1

What is the volume of a prism whose cross section has an area of 25 cm^2 and which is 12 cm high?

Solution: Volume = B x h ----------------B = 25 cm^2; h = 12 cm

Substitute these into the formula, we have

$V = (25\ cm^2)(12\ cm)$

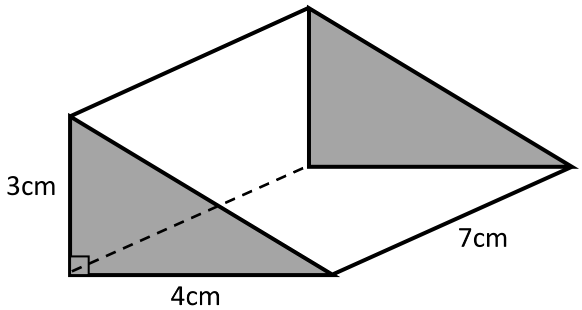
$\mathbf{V = 300\ cm^2}$

Usually the area of the cross-section is not given. Therefore, it must be calculated first before we can work out the volume. Be very careful to use the face that has a uniform cross-section throughout the solid when working out the cross-sectional area.

Let us do an example.

Example 2

Work out the volume of the following triangular prism.



Solution:

(i) Find the area of cross-section (triangular face).

base = 4 cm, height = 3 cm

Area of cross-section $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 4 \times 3$

$=$ **6 cm^2**

(ii) Calculate the volume of the prism.

Area of cross-section = 6 cm^2, length = 7 cm

Volume = (Area of cross-section) **x** length

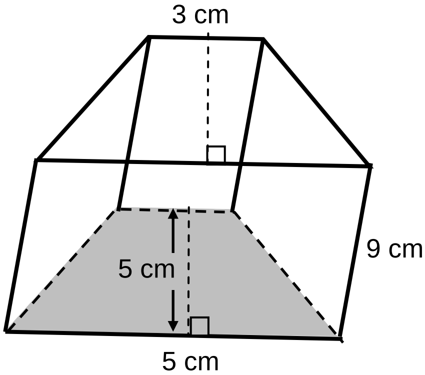
$= 6 \times 7$

$=$ **42 cm^3**

Therefore, the volume of the prism is 42 cm^3.

Example 3

Find the volume of the trapezoidal prism whose length is 9 cm, height is 5 cm and the lengths of parallel sides are 5 cm and 3 cm as shown.



Solution:

(i) Find the area of cross-section or base (trapezoidal face).

base = 4 cm, height = 3 cm

Area of cross-section $= \frac{1}{2}(a + b) \times h$

$= \frac{1}{2}(3 \text{ cm} + 5 \text{ cm}) \times 5 \text{ cm}$

$= \frac{1}{2}(8 \text{ cm}) \times 5 \text{ cm}$

$=$ **20 cm^2**

(ii) Calculate the volume of the prism.

Area of cross-section = 20 cm^2, length = 9 cm

Volume = (Area of cross-section) **x** length

= 20 cm^2 **x** 9 cm

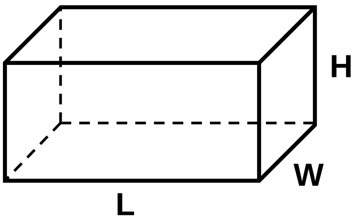
= **180 cm^3**

Therefore, the volume of the prism is 180 cm^3.

We will now learn to calculate the volume of a rectangular prism.

As you learnt earlier, a **rectangular prism** is the simplest form of prism which has all six faces shaped like rectangles. This is understood that the base or the cross section is a rectangle.

The volume of a rectangular prism is the amount of space occupied by the prism. If we know the **length**, **width** and **height** of a rectangular prism (**cuboid**), we can find its volume.



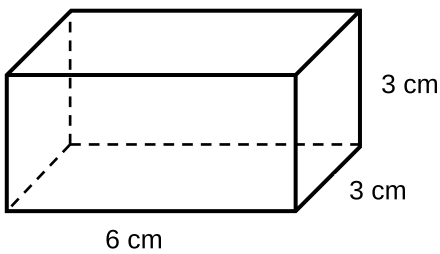
The formula for calculating the volume of a rectangular prism is:

Volume = B x H --------- B = L x W

$$\boxed{\mathbf{V = L \times W \times H}}$$

Example 4

Find the volume of the following rectangular prism or cuboid.



Solution:

Volume = L × W × H

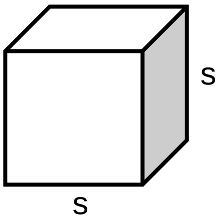
= 6 cm × 3 cm × 3 cm

= 54 cm^3

Now we will learn to calculate the volume of a cube.

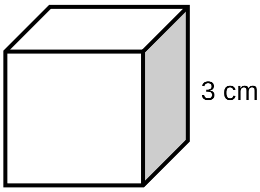
Since a cube has sides that are all the same size, this is a very easy volume formula to remember. We are going to find the area of the bottom of the cube and multiply by the height. So, that is length x width x height. Since the length, width and height are all the same dimensions, we can cube the length of the side. Hence, the formula for the volume of a cube is:

Volume = s x s x s
$V = s^3$



Example 5

Find the volume of a cube whose side length is 3 cm.



$V = s^3$

V = (3 cm)(3 cm)(3 cm)

$V = 27 \text{ cm}^3$

So far we have learnt how to calculate the volume of prisms.

This time we will learn how to work out:

1. The area of cross-section or base, given the volume and height or length.
2. The height or length, given the volume and area of cross-section or base.

To calculate the area of the cross section of a prism given the volume and the height, divide the given volume by the height of the prism.

Using the formula: **V = B x H**

If **V** represents the volume of prism, **B** the area of cross section or base and **H** the height, then by changing the subject of the formula, we have

$$\mathbf{B = \frac{V}{H}}$$

In the same way

To calculate the height of a prism given the volume and the height, divide the given volume by the area of the cross section or base of the prism.

$$\mathbf{H = \frac{V}{B}}$$

Look at the examples on the next page.

Example 6

A tank in the shape of a triangular prism has a cross-sectional area of 300 m^2 and a volume of 2850 m^2. Find the height of the tank.

Solution:

$V = 2850\ m^3$, $A = 300\ m^2$, height = h

$$h = \frac{V}{A}$$

$$= \frac{2850}{300}$$

$$= \mathbf{9.5\ m}$$

That is, the height of the tank is 9.5 m.

Example 7

Find the area of cross-section of the prism below given that its volume is 231 m^3 and the height is 11 m.

Solution: Volume = 231 m^3
Height = 11 m

$$B = \frac{V}{H}$$

$$B = \frac{231\ m^3}{11\ m}$$

$$\mathbf{= 21\ m^2}$$

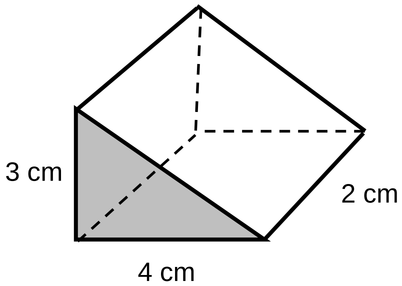
That is, the cross section area of the prism is 21 m^2.

NOW DO PRACTICE EXERCISE 16

Practice Exercise 16

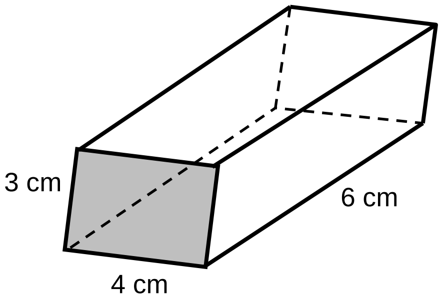
1. Calculate the volume of the following prisms.

(a)



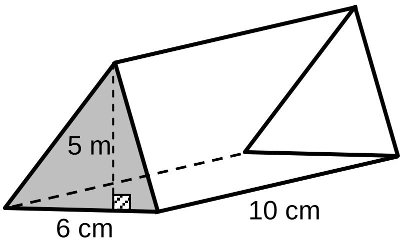
Answer: __________

(b)



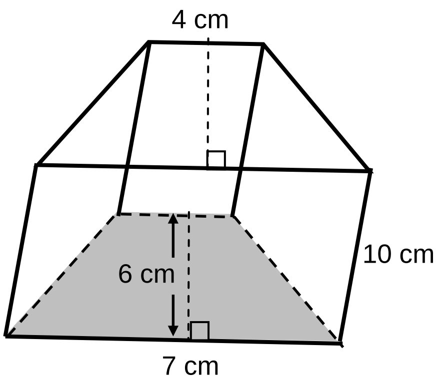
Answer: __________

(c)



Answer: __________

(d)



Answer: __________

2. Find the height of the prisms that have the following measurements.

(a) Volume = 250 m^3
Area of cross-section = 25 m^2

Answer: __________

(b) Volume = 280 cm^3
Area of cross-section = 40 cm^2

Answer: __________

---

3. Calculate the area of cross-section of the prisms with the following measurements.

(a) Volume = 567 cm^3
Height = 9 cm

Answer: __________

(b) Volume = 48.4 m^3
Height = 8 m

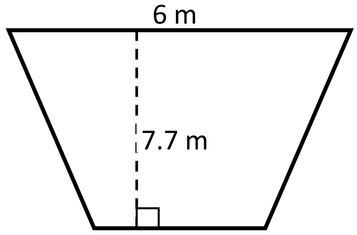
Answer: __________

---

4. A triangular prism-shaped container has a total volume of 250 cm^3 and a cross-sectional area of 25 cm^2.
Find the height of the prism.

Answer: __________

5. Below is a diagram showing the base of a trapezoidal prism-shaped water container



(a) Calculate the area of the cross-section.

Answer: ___________

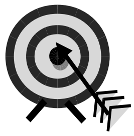
(b) Water is poured into the container, to a height of 13.8 m.
Find the volume of the water inside.

Answer: ___________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 17: Surface Area and Volume of Pyramids

You learnt to solve the surface area and volume of prisms in the previous lessons.



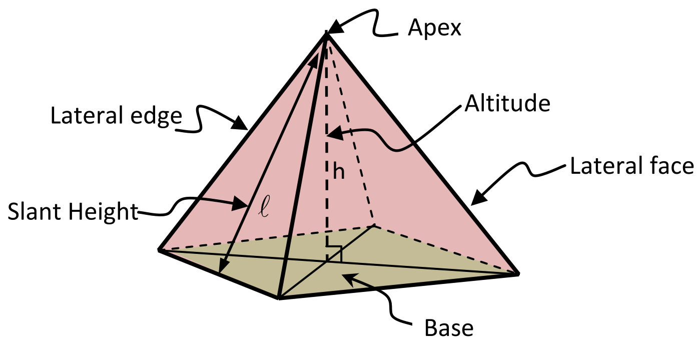
In this lesson, you will:

- define a pyramid and identify its features
- calculate the surface area and volume of pyramids.

Let us revise some of the work we did in Grade 8.

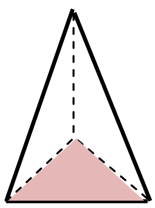
A pyramid is a solid that has a **base** in the shape of a polygon, each vertex of which is joined to a single point in a plane other than that of the base**.** This point is known as the **vertex** of the pyramid which is also called the **apex** of the pyramid. The sides of the pyramids, known as its **lateral faces**, are all triangular in shape and meet at the vertex. The segments where the lateral faces meet are called the **lateral edges**. The **altitude** (h) of the pyramid is the segment perpendicular from the vertex to the base. The **slant height** (ℓ)of the pyramid is the segment perpendicular from the vertex to the base of a lateral face.

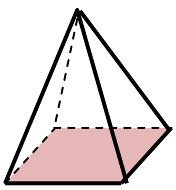
Here is a pyramid with its parts.

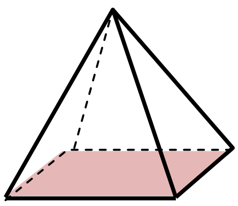


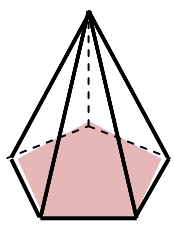
There are many types of pyramids, and they are named after the shape of their bases.

Below are the types of pyramids.


Triangular pyramid

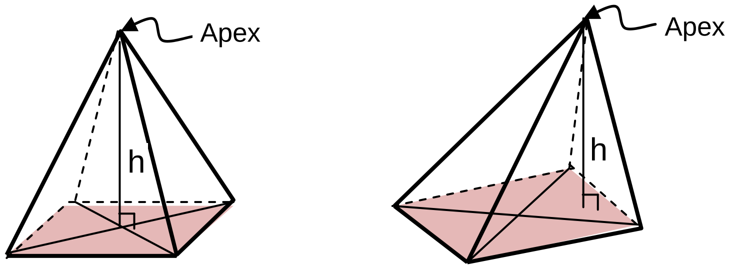

Square pyramid


Rectangular pyramid


Pentagonal pyramid

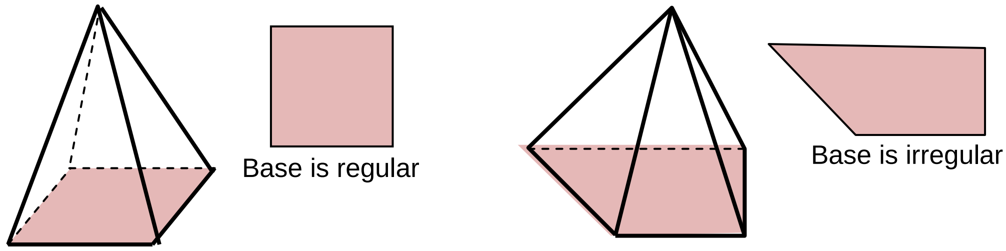
- If the apex or vertex is directly above the centre of the base, then it is a **right pyramid.** Otherwise it is an **oblique pyramid**.

  See figures on the next page.



Right pyramid Oblique pyramid

- If the base of the pyramid is a regular polygon, then it is a **regular pyramid**. Otherwise it is an **irregular pyramid.**



Regular pyramid Irregular pyramid

Surface Area and Volume of a Pyramid

We will now learn to work out the surface area and volume of a pyramid.

As we have learnt in Grade 8, pyramids, just like the other solids you have looked at, have lateral surface areas and a total surface area.

The Surface Area of a pyramid has two parts:

- the area of the base (the **Base Area**), and
- the area of the side faces (the **Lateral Surface Area**).

The **Base Area** depends on the shape. There are different formulas for the triangle, square, etc.

For the **Lateral Surface Area** when all the side faces are the same:

- Just multiply the perimeter by the "slant height" and divide by 2. This is because the side faces are always triangles and the triangle formula is *"base times height divided by 2"*

In symbols we have $\textbf{LSA} = \frac{1}{2}\textbf{P}\boldsymbol{\ell}$

Where **LSA** is the lateral surface area

P is the perimeter of the base

$\boldsymbol{\ell}$ is the slant height

- But if the side faces are different (such as in an "irregular" pyramid) then add up the area of each triangular shape, to find the lateral surface area.

So, to find the Total Surface Area of a Pyramid, the general formula is:

When all side faces are the same,

Total Surface Area (TSA) = [Base Area] + $^1/_2$ × Perimeter × [Slant Length]

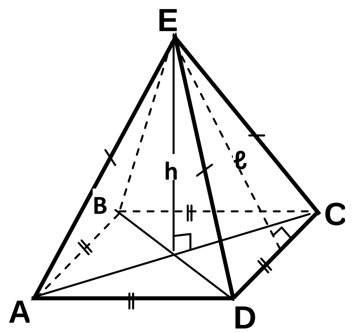
$$\boxed{\mathbf{TSA = B + \frac{1}{2}P\ell}}$$

When side faces are different

Total Surface area = [Base Area] + [Lateral Surface Area]

Example 1

The base of the regular pyramid below is a square. Given that AD = 12 cm, DE = 6 cm, h = 4 cm and ℓ = 5 cm.



a) Find the lateral surface area of the regular pyramid.

b) Find the total surface area of the regular pyramid.

Solution:

a) Find the lateral surface area using the formula LSA = $\frac{1}{2}P\ell$

Since the base is a square, the perimeter (P) = 4 x AD

P = 4 x 12 cm

= 48 cm

Write the formula: LSA = $\frac{1}{2}P\ell$

Substitute: LSA = $\frac{1}{2}$ x 48 cm x 5 cm

= 120 cm^2

Therefore the lateral surface area of the pyramid is 120 cm^2.

b) Find the total surface area.

First find B (Area of the square base). $B = (AD)^2$

Substitute: $B = (12\text{ cm})^2$

$= 144\text{ cm}^2$

Use the formula: TSA = B + LSA

Substitute: $B = 144\text{ cm}^2$ and $LSA = 120\text{ cm}^2$ to the formula

Hence, we have $TSA = 144\text{ cm}^2 + 120\text{ cm}^2$

$\mathbf{= 264\text{ cm}^2}$

Therefore, the total surface area of the pyramid is 264 cm^2.

Let us now learn to work out the volume of a pyramid.

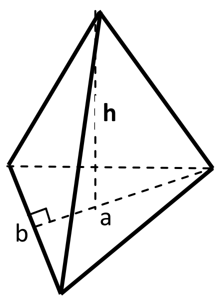
We learnt that:

The volume of a pyramid is equal to one third of its base area multiplied by it height.

$$\boxed{V = \frac{1}{3}Bh}$$

Since, there are different types of pyramids the volume of a pyramid depends on the area of base and its height.

So, for triangular pyramid: $$V = \frac{1}{3}\left[\left(\frac{1}{2}ab\right)h\right]$$



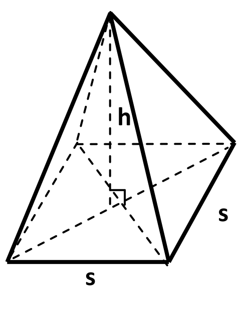
where V= volume of the pyramid

a = altitude of the triangle base

b = base of the triangle base

h = height of the pyramid

For a Square pyramid: $$V = \frac{1}{3}s^2h$$

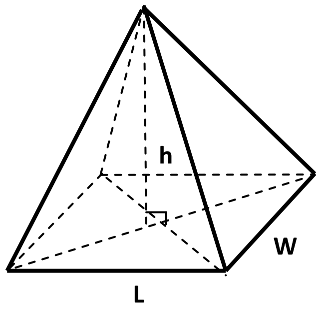


where V = the volume of the pyramid

s = a side of the square base

h = the height of the pyramid

For rectangular pyramids: $\mathbf{V = \frac{1}{3}(L \times W) \times h}$



where V = the volume of the pyramid

L = the length of the rectangle base

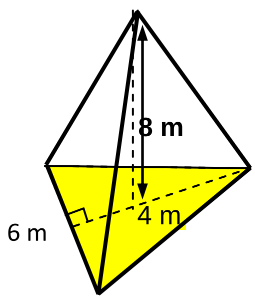
W = the width of the rectangle base

h = the height of the pyramid

Now let us look at the following examples.

Example 1

A triangular pyramid has a height of 8 meters. If the triangle has a base of 6 meters and a height of 4 meters, what is the volume of the pyramid?



Solution:

Notice that in this example, you are dealing with two different heights: the height of the pyramid and the height of the triangle base. Avoid mixing the first with the latter.

To find the volume of the triangular pyramid use the formula: $V = \mathbf{\frac{1}{3}\left[\left(\frac{1}{2}ab\right)h\right]}$

Substitute: a = 4 m, b = 6 m and h = 8 m

Hence we have: $V = \frac{1}{3}\left[\frac{1}{2}(4\text{ m})(6\text{ m})(8\text{ m})\right]$

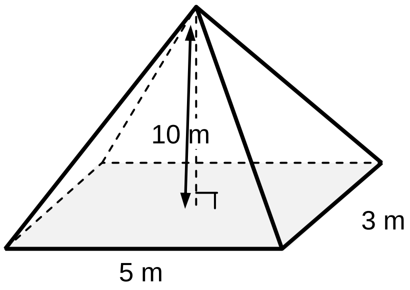
$= \frac{1}{3}(96\text{ m}^3)$

$\mathbf{= 32\text{ m}^3}$

Therefore, the volume of the triangular pyramid is 32 m^3.

Example 2

A rectangular pyramid has a height of 10 meters. If the sides of the base measure 3 meters and 5 meters, what is the volume of the pyramid?



Solution:

To find the volume of the rectangular pyramid use the formula: $\mathbf{V = \frac{1}{3} (L \times W) \times h}$

Substitute: L = 5 m, W = 3 m and H = 10 m

Hence, we have:

$$V = \frac{1}{3}(L \times W) \times h$$
$$= \frac{1}{3}(5\text{ m})(3\text{ m})(10\text{ m})$$
$$= \frac{1}{3}(150\text{ m}^3)$$
$$\mathbf{= 50\text{ m}^3}$$

Therefore, the volume of the triangular pyramid is 50 $\mathbf{m^3}$.

Example 3

A square pyramid has a height of 9 meters. If a side of the base measures 4 meters, what is the volume of the pyramid?

Solution:

To find the volume of a Square pyramid, use the formula: $\mathbf{V = \frac{1}{3} s^2h}$

Since the side of the base (s) = 4 m and the height of the pyramid (h) = 9 m

Substituting these in the formula, we have:

$$V = \frac{1}{3}s^2h$$
$$= \frac{1}{3}(4\text{ m})^2(9\text{ m})$$
$$= \frac{1}{3}(16\text{ m}^2)(9\text{ m})$$
$$\mathbf{= 48\text{ m}^3}$$

Therefore, the volume of the square pyramid is 48 $\mathbf{m^3}$.

NOW DO PRACTICE EXERCISE 19

Practice Exercise 17

1. Find the lateral surface area of a regular pyramid with triangular base if the base measures 8 cm and the slant height is 5 cm.

2. Find the total surface area of a regular square pyramid if each edge of the base measures 12 cm, the slant height is 10 cm and the altitude is 6 cm.

3. Find the volume of a square pyramid measuring 10 cm by 10 cm by 9 cm high.

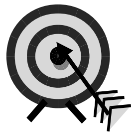
4. Find the volume of a triangular pyramid with base area 15 cm^2 and height 5 cm.

5. Find the volume of a square pyramid with base length 8 cm and height 3 cm.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4

Lesson 18: Surface Area and Volume of Cylinders

You learnt to compute the surface area and volume of pyramids in the previous lessons.



In this lesson, you will:

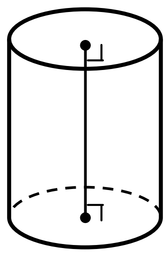
- define a cylinder and identify its features
- calculate the surface area and volume of pyramids.

Let us recall what we learnt about cylinders in our Grade 8 Mathematics. We learnt that some solids are neither prisms nor pyramids. One of these is the cylinder.

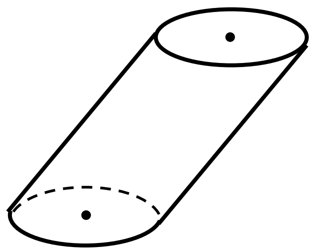
Here again is the definition of a cylinder.

A cylinder is a 3-dimensional shape that has two circular ends or bases that are parallel and congruent and are connected by a curved surface.

When the centres of the bases are aligned directly one above the other, it is called a **right cylinder.** Otherwise**,** when the centres of the bases of a cylinder are not aligned directly one above the other, it is called an **oblique cylinder.**



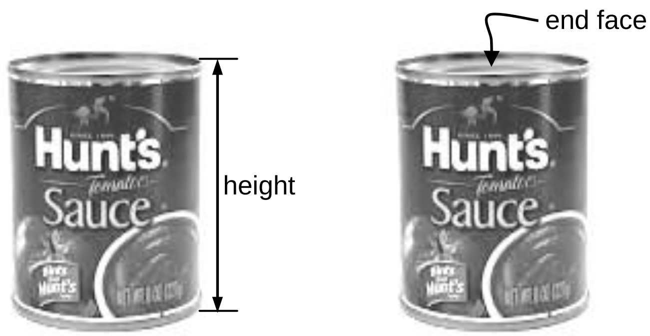
Right cylinder



Oblique cylinder

In this discussion, we will only consider finding the surface area and volume of a right cylinder.

Food cans like these have circular ends so we call them **right cylinders**.

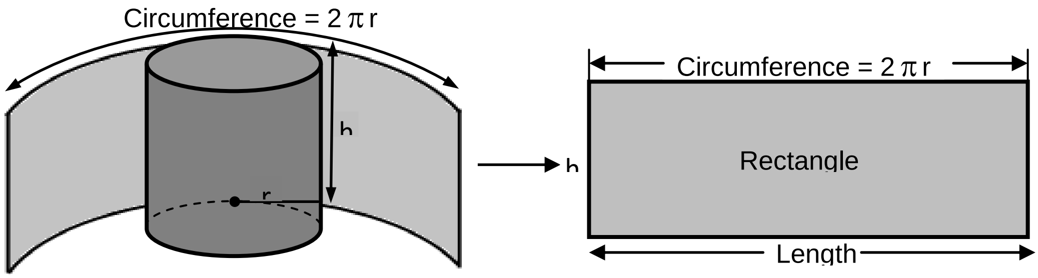


Surface Area of a Cylinder

All of the solids we learnt so far, the surface area have been made of flat or plane surfaces.

With the cylinder, this is no longer the case. The cylinder's surface area is made up of a curved surface and two circles.

Here is a cylinder.



Curved surface area of a Cylinder

To calculate the area of the curved surface, imagine that the cylinder is pictured as a food can with label. Its lateral area is the area of the label. If the label is carefully peeled off, the label becomes a rectangle. **The area of this rectangle would be the same as the area of the curved surface.**

The length of the rectangle is the same as the circumference (**C**) of a base of the cylinder. The width of the rectangle is the same as the height (**h**) of the can.

Therefore, the area of the curved surface is the product of the circumference of the base and the height of the can.

To find the curved surface area, we must find the circumference of the base since it represents the length of the curved surface.

$$\text{Circumference of base} = \mathbf{2\pi r}$$
$$\mathbf{C = 2\pi r}$$

Therefore, Area of curved surface (CSA) = circumference of base x height of the can

$$\text{Area of curved surface (CSA)} = C \times h = (2\pi r \times h)$$

$$\boxed{\mathbf{CSA = C \times h \quad or \quad 2\pi rh}}$$

Formula for Lateral Surface Area of a Cylinder

You have to calculate the area for both the circles, and the cylindrical part in between. The cylindrical part, in between, is simply a rectangle that has been rolled up.

The area of the two circles on either end is known as the **lateral surface area** of a cylinder.

To find the area of a circle, we need to know its **radius** (r) or **diameter** (d). The radius is the distance from the centre of a circle to the circumference. The diameter is twice the radius.

The formula for the area of a circle is $\pi \mathbf{r^2}$. The value π, or ***pi***, is a mathematical number calculated as $\frac{22}{7}$, or 3.14.
Therefore, after using the formula and getting a number as its result, to find the total lateral surface area we simply multiply this number by 2.

Hence, the formula for the lateral surface area (LSA) is equal to the areas of the two circles.

Lateral Surface Area = 2 x πr^2

$$\boxed{\mathbf{LSA} = 2\pi r^2}$$

Equation for Total Surface Area of a Cylinder

All we need to do is add up these two amounts, that is, the curved surface area and the lateral surface area.

Total Surface Area = Area of curved surface + Lateral surface area

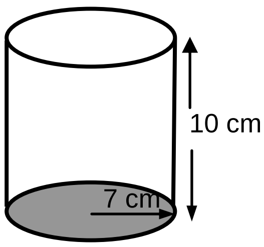
$$\mathbf{TSA = CSA + LSA}$$

$$\mathbf{TSA} = 2\pi rh + 2\pi r^2$$

$$\boxed{\mathbf{TSA} = 2\pi r(h + r)}$$

Example 1

Find the total surface area of the cylinder below with radius 5 cm and height 10 cm. Use Pi = $\frac{22}{7}$.



Solution: r = 7, and h = 10.

First, find the lateral surface area (LSA): LSA = $2\pi r^2$

$= 2 \times \frac{22}{7} \times 7 \times 7$

$\mathbf{= 308\ cm^2}$

Second, find the curved surface area (CSA): CSA = $2\pi rh$

$= 2 \times \frac{22}{7} \times 7 \times 10$

$\mathbf{= 440\ cm^2}$

Finally, add the two amounts to find the total surface area.

Total surface area (TSA) = 308 cm^2 + 440 cm^2

$\mathbf{= 748\ cm^2}$

Therefore, the total surface area of the cylinder is 748 cm^2.

Alternately, you can apply the formula for total surface area directly as well.

Total surface area = 2πr(h + r)

$= 2 \times \frac{22}{7} \times 7(10 + 7)$

$= 2 \times \frac{22}{7} \times 7 \times 17$

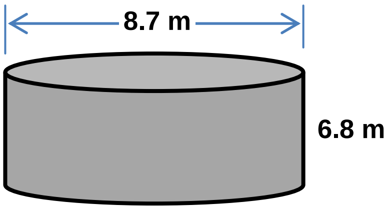
$\mathbf{= 748\ cm^2}$

This was the formula for calculating the cylinder's total surface area, and this can come in handy many times.

Let us look at another example.

Example 2

For the cylinder given at the right, find:



(a) the curved surface area(CSA)

(b) the area of the circular ends(LSA)

(c) the total surface area (TSA).

Use $\pi = 3.14$. Give the answers to the nearest metre.

Solution: r = 3.45 m, h = 6.8 m

(a) to find the curved surface area use the formula: **CSA = 2πrh**

CSA = 2 x 3.14 x 4.35 x 6.8

$\mathbf{= 186\ m^2}$

(b) To find the Area of circular ends LSA) used the formula: **LSA = 2πr²**

LSA = 2 x 3.14 x $(4.35)^2$

$\mathbf{= 119\ m^2}$

(c) To find the Total surface area(TSA) add the two areas obtained in (a) and (b).

TSA = 186 m^2 + 119 m^2

$\mathbf{= 305\ m^2}$

or use the formula: **TSA= 2πr(h + r)**

= 2 x 3.14 x 4.35 x (6.8 + 4.35)

= 2 x 3.14 x 4.35 x 11.15

= 304.5957

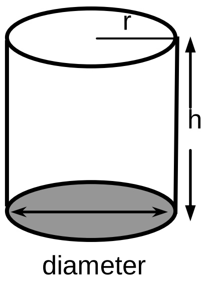
$\mathbf{= 305\ m^2}$

Formula for Volume of a Cylinder

Since a cylinder has a constant cross-section, like a prism, we use the same formula for working out the volume of a cylinder as we did for working out the volume of a prism.

That is: **Volume = Area of cross-section x height (length)**

Since the cross-section of a cylinder is a **circle**, the formula for calculating the area of a circle (πr^2) is used to find the area of the cross-section.

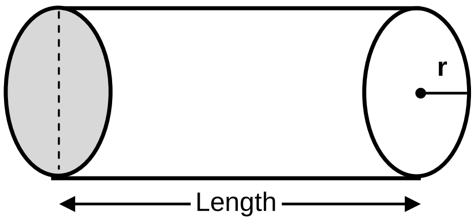


Therefore, the formula for volume of a cylinder is:

Volume = (Area of cross-section) x height

$$\boxed{V = (\pi r^2) \times h}$$

If the cylinder lies on its side like this:

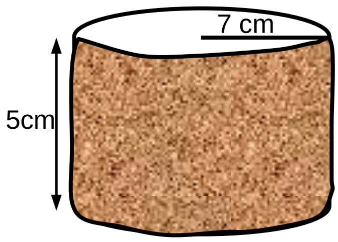


Then:

Volume = Area of cross-section x Length

$$\boxed{V = (\pi r^2) \times l}$$

Example 1 Calculate the volume of the cylinder below. Use π =



Solution: radius = 7 cm

Area of cross-section = πr^2

$= \frac{22}{7} \times 7 \times 7$

$=$ **154 cm²**

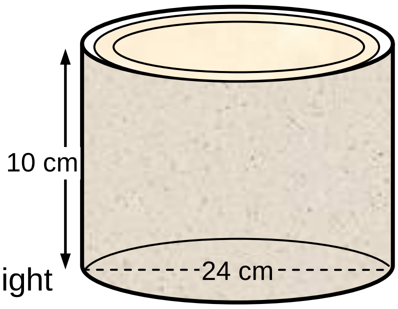
Volume = (Area of cross-section) x height

$= 154 \times 4$

$=$ **616 cm³**

Therefore volume of the cylinder is 616 cm^3.

Example 2 A cylindrical tin 10 cm high has a diameter of 24 cm. Calculate its volume. Use $\pi = 3.14$



Solution: diameter = 24 cm, so, radius = 12 cm

Area of cross-section = πr^2
$= 3.14 \times 12 \times 12$
$= \mathbf{452.16\ cm^2}$

Volume of tin = (Area of cross-section) **x** height
$= 452.16 \times 10$
$= \mathbf{4521.6\ cm^3}$

Therefore volume of the tin is $4521.6\ cm^3$.

The next example involves conversion of metric units.

Example 3 A cylindrical water tank has a diameter of 7 metres. It contains water to a depth of 2.5 metres. Find:

(a) The volume of water inside the tank, in cubic metres. Use $\pi = \frac{22}{7}$.

Solution: diameter = 7 m, so, radius = 3.5 m

Area of cross-section = πr^2
$= \frac{22}{7} \times 3.5 \times 3.5$
$= \mathbf{38.5\ m^2}$

Volume of water = (Area of cross- section) x height
$= 38.5 \times 2.5$
$= \mathbf{96.25\ m^3}$

Therefore, the volume of water in cubic metres is $96.25\ m^3$.

(b) The volume of liquid is usually measured in litres or millilitres. Find the volume of the water in litres.

Solution:
Volume of water = $96.25\ m^3$

$96.25\ m^3 = 96.25 \times 1000$ L ....$1\ m^3 = 1000$ L
= **96 250 L**

There are altogether 96 250 litres of water in the tank.

NOW DO PRACTICE EXERCISE 18

Practice Exercise 18

1. For the cylinders given below, find:

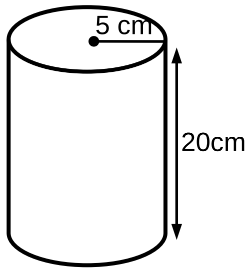
   (a) the curved surface area(CSA)

   (b) the area of the circular ends(LSA)

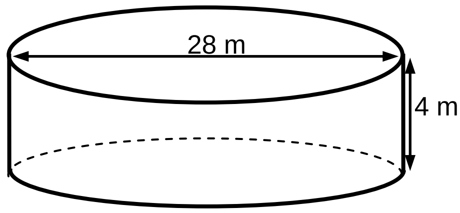
   (c) the total surface area (TSA).

Use π = 3.14. Give the answers to the nearest metre.

(i)



(ii)



2. Find the total surface area of a cylinder with a radius of 4 cm and height of 6 cm. Use $\pi = \frac{22}{7}$. (Hint: TSA = $2\pi r^2 + \pi rh$).

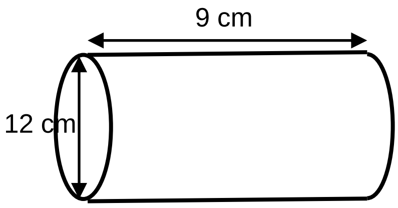
3. Mary is wrapping a can of paint as a gag gift for a friend. If the can is 11 cm high and has a diameter of 8 cm, how many square inches of wrapping paper will she use in completely covering the can? (Let pi = 3.14).

4. Complete the table below concerning the volume of cylinders.

	Area of cross-section	Height	Working out	Volume
(a)	10 cm^2	5 cm		
(b)	2.8 m^2	2 m		
(c)	305 cm^2	87 cm		

---

5. Study this cylinder.



(a) Find the area of its cross-section. Use $\pi = 3.14$
Write your answer to the nearest square centimetre.

Answer: ____________

(b) Find the volume.
Write your answer to the nearest cubic centimetre.

Answer: ____________

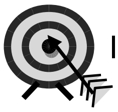
(c) Does it hold more than 1 litre or less?
(1 L = 1000 cm^3)

Answer: ____

CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 19: Surface Area and Volume of Cones

You have learnt to find the surface area and volume of a cylinder in the last lesson..



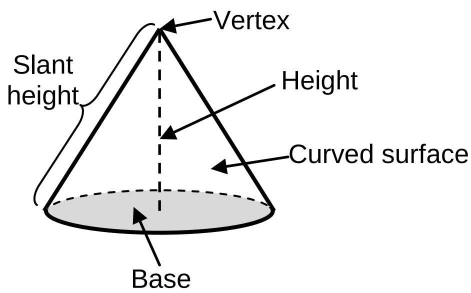
In this lesson, you will:

- define cone and identify its features
- calculate the surface area and volume of a cone.

In Gr 8, you were introduced to the meaning of a cone.

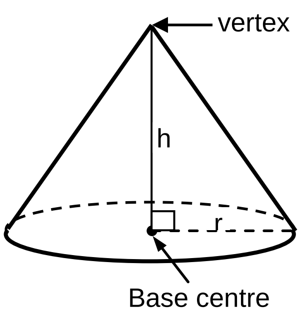
Let us recall what you learnt.

A cone is a special solid figure with a plane circular base bounded by a conical surface. The height of the cone is the perpendicular distance from the base to the vertex. The slant height is the length of the inclined portion that joins the vertex with the base.

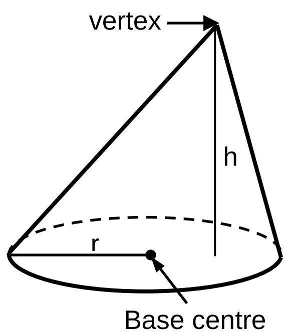


A cone is called a **right cone**, if the vertex of the cone is vertically above the centre of the base. When the vertex of the cone is not vertically above the centre of the base, it is called an **oblique cone**.

The following diagram shows a right cone and an oblique cone.



Right Cone

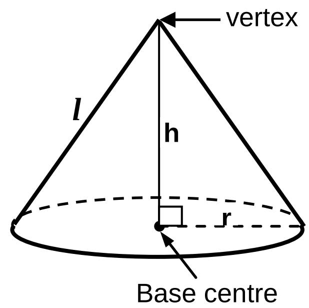


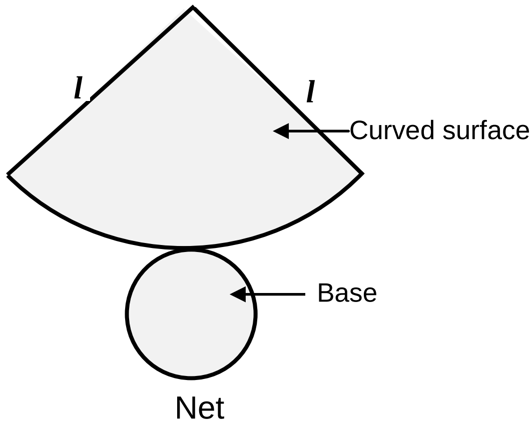
Oblique Cone

In common usage, cones are assumed to be right and circular. Its vertex is vertically above the centre of the base and the base is circular.

The **net** of a cone consists of the following two parts:

- a circle that gives the base; and
- a sector that gives the curved surface





The Total Surface Area of a Cone

There are two components that you need to calculate while determining the total surface area (TSA) of a cone.

- Area of the circular base of the cone
- Area of the Curved or Lateral surface of the cone

TSA = Area of the Circular Base of Cone + Area of Curved Surface of Cone

The first step in finding the surface area of a cone is to measure the radius of the circular base of the cone. The next step is to find the area of the circle, or base. The area of a circle is 3.14 times the radius squared (πr^2).

Area of the Circular Base of Cone (A) = πr^2

Now, you will need to find the curved surface area of the cone itself. In order to do this, you must measure the side (slant height) of the cone. Make sure you use the same form of measurement as the radius.

You can now use the measurement of the side (slant height) to find the curved surface area of the cone. The formula for the curved surface area of a cone is 3.14 times the radius times the slant height ($\pi r l$).

Area of Curved Surface of Cone (CSA) = $\pi r l$

So, the total surface area of a cone is determined by adding the two areas:

TSA = Area of the Circular Base of Cone + Area of Curved Surface of Cone

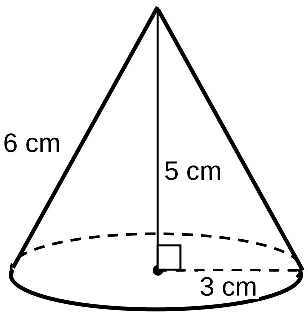
$\boxed{\text{TSA} = \pi r^2 + \pi r l}$ **or** $\boxed{\text{TSA} = \pi r(r + l)}$

Where **r** is the radius of the circle and l is the slant height of the cone.

Now let us look at the examples on the next page.

Example 1

A cone has a radius of 3 cm and height of 5 cm. Find total surface area of the cone if the cone has a slant height of 6 cm. Use π = 3.14.



Solution:

To begin with we need to find the area of the circular base of the cone, which is determined by the formula:

Area of circular Base = $\boldsymbol{\pi r^2}$

$= 3.14 \times (3 \text{ cm})^2$

$= 3.14 \times 3 \text{ cm} \times 3 \text{ cm}$

$= 28.26 \text{ cm}^2$

Next, we need to find the area of the curved surface of the cone, which is determined by the formula:

Area of Curved Surface = $\boldsymbol{\pi r l}$

$= 3.14 \times 3 \text{ cm} \times 6 \text{ cm}$

$= 56.52 \text{ cm}^2$

And the total surface area of the cone is:

TSA = $\boldsymbol{\pi r^2 + \pi r l}$

$= 28.26 \text{ cm}^2 + 56.52 \text{ cm}^2$
$= 84.78 \text{ cm}^2$

Example 2

The slant height of a cone is 26 cm, and the radius of the base is 10 cm. Find the curved surface area of the cone.

Solution: l = 26 cm; r = 10 cm

CSA = $\boldsymbol{\pi r l}$

$= 3.14 \times 10 \text{ cm} \times 26 \text{ cm}$

$= 81.64 \text{ cm}^2$

Therefore, the curved surface area is **81.64 cm^{2}**.

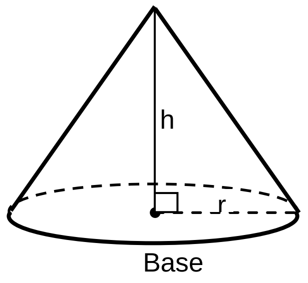
We are now going to look at the volume of a cone.

The volume of a cone

In Grade 8, you learnt that the volume of a cone is one-third the volume of a cylinder.

So if the volume of a cylinder is $\mathbf{V = \pi r^2 h}$

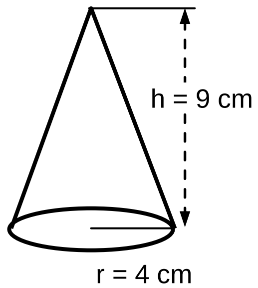
For the cone, we have $\mathbf{V = \frac{1}{3} \pi r^2 h}$



Where V = volume of the cone
r = radius of the base
h = height of the cone

Let's find the volume of the cone in the example below.

Example 1



Find the volume of the cone at the right.

Solution:

We can substitute the values into the volume formula.

When we perform the calculations, we find that the volume is 150.72 cubic centimetres.

Here is the working out.

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}(3.14)(4\text{ cm})^2(9\text{ c m})$

$V = \frac{1}{3}((3.14)(16\text{ cm}^2)(9\text{ cm})$

$V = \frac{1}{3}(50.24\text{ cm}^2)(9\text{ cm})$

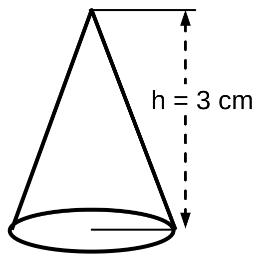
$V = \frac{1}{3}(452.16\text{ cm}^3)$

$\mathbf{V = 150.72\text{ cm}^3}$

Example 2

Calculate the volume of a cone if its radius is 2 cm and its height is 3 cm.

Solution: $V = \frac{1}{3}\pi r^2 h$



$V = \frac{1}{3} \times 3.14 \times 2^2 \times 3$

$V = \frac{1}{3} \times 3.14 \times 4 \times 3$

$V = \frac{1}{3} \times 37.68$

$V = 12.56 \text{ cm}^3$

Example 3

Find the volume of a cone whose base has a diameter of 5 cm and whose height is 9 cm. Round your answer to the nearest centimetre.

Solution: $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3} \times 3.14 \times 2.5^2 \times 9$

$V = \frac{1}{3} \times 3.14 \times 6.25 \times 9$

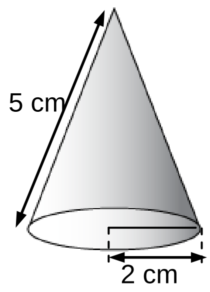
$V = \frac{1}{3} \times 176.625$

$V = 59 \text{ cm}^3$

NOW DO PRACTICE EXERCISE 18

Practice Exercise 19

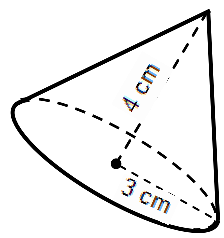
1. Find the surface area of a cone with a base of a circle with a radius of 2 cm and has a slant height of 5 cm. HINT: **TSA =** $\boldsymbol{\pi r^2 + \pi r l}$



2. Find the surface area of a cone with radius 6 cm and height 11 cm.. Round off your answer to the nearest square centimetre.

3 Calculate the volume of a cone if the height is 12 cm and the radius is 7 cm.

4. Find the surface area of the following solid cone with a base of radius 3 cm and a height of 4 cm. Express answer in terms of $\boldsymbol{\pi}$.



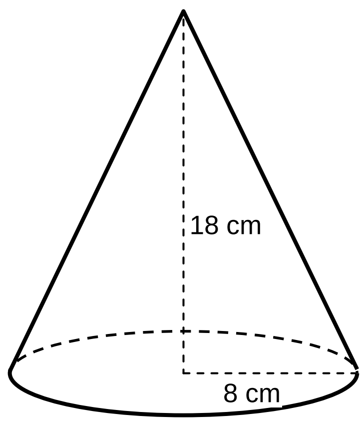
5. Calculate the surface area of a cone having the radius of the base as 4 cm and the slant height is 8 cm?

---

6. Calculate the surface area of a cone having the diameter of the base as 10 cm and the slant height is 6 cm?

---

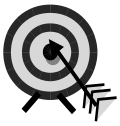
7. Find the volume of the cone shown. Round off your answer to the nearest tenth of a cubic centimetre.



CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4

Lesson 20: Surface Area and Volume of Spheres

You learnt how to find the surface area and volume of a cone in the last lesson.



In this lesson, you will:

- define a sphere and identify its features
- calculate the surface area and the volume of a sphere.

One of the most commonly occurring shapes in everyday life is the **sphere**.
For example, a shot put ball (a heavy iron ball) is a solid sphere and a tennis ball or basketball is a hollow ball.

Shot put ball Billiard ball

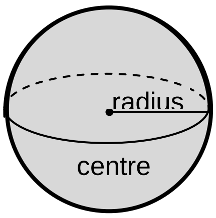
SOLID SPHERES

Tennis ball Basketball ball

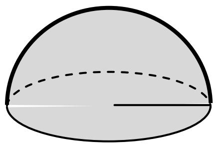
HOLLOW SPHERES

But what is a sphere?.

A **sphere** is a solid with all its points the same distance from a fixed point called the center. The distance is known as the radius of the sphere. The maximum straight distance through the center of a sphere is known as the diameter of the sphere. The diameter is twice the radius. One half of a sphere is called a **hemisphere**.



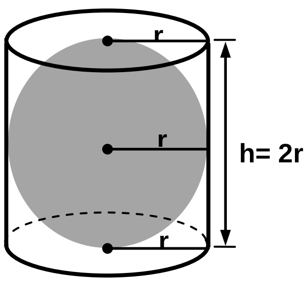
SPHERE



HEMISPHERE

Surface area of a sphere

Archimedes, a Greek mathematician and inventor, discovered that a cylinder that circumscribes a sphere, as shown in the following diagram, has a curved surface area (**CSA**) equal to the surface area (**S**) of a sphere.



For a sphere of radius **r:**

Surface area of a sphere = Curved surface area of a cylinder

$$S = 2\pi rh$$

$$S = 2\pi r \times 2r$$

Hence, the formula for the surface area of a sphere is:

$$\boxed{S = 4\pi r^2}$$

If you are given the diameter (d), remember to first divide the diameter by 2 to get the radius before using the formula.

Example 1

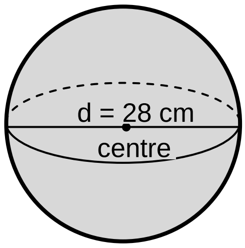
Find the surface area of a sphere of diameter of 28 cm using $\pi = \frac{22}{7}$.

Solution: $d = 28$ cm

$r = \frac{1}{2}d$

$= \frac{1}{2}(28 \text{ cm})$

$r = 14$ cm



Now, $S = 4\pi r^2$

$S = 4(\frac{22}{7})(14 \text{ cm})^2$

$S = 4(\frac{22}{7})(196 \text{ cm}^2)$

$S = 2464 \text{ cm}^2$

So, the surface area of the sphere is 2464 cm^2.

Example 2

Calculate the surface area of a sphere with radius 3.2 cm. Use $\pi = 3.14$.

Solution: $r = 3.2$ cm

$S = 4\pi r^2$

$= 4(3.14)(3.2 \text{ cm})^2$

$= 4 \times 3.14 \times 10.24 \text{ cm}^2$

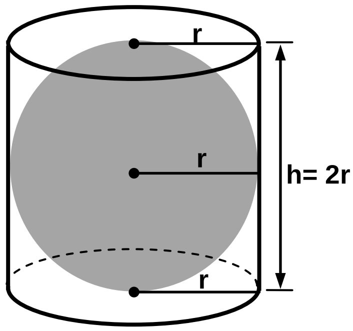
$= 128.6 \text{ cm}^2$

So, the surface area of the sphere is 128.6 cm^2.

On the next page, you will learn to work out the volume of a sphere.

Volume of a sphere

The formula for the volume of a sphere was derived by Archimedes. He showed that the volume of sphere is $\mathbf{\frac{2}{3}}$ of a circumscribed cylinder.



Since the volume of a cylinder = $\pi r^2 h$

It can be shown that for a sphere of radius **r**:

Volume of the sphere = $\frac{2}{3} \times \pi r^2 h$

$= \frac{2}{3} \times \pi r^2 \times 2r$

$= \frac{4}{3} \pi r^3$

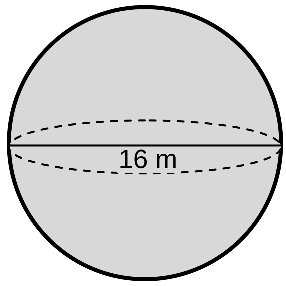
Therefore, formula for volume of the sphere is given by:

$$\boxed{V = \frac{4}{3} \pi r^3}$$

As with the surface area, if you are given the diameter (d), remember to first divide the diameter by 2 to get the radius before using the formula.

Example 1

Find the volume of the sphere at the right.
Round off your answer to the nearest cubic meter.



Solution:

From the figure, the diameter of the sphere is 16 m.

Solution: $d = 16$ m

$r = \frac{1}{2} d$

$= \frac{1}{2}(16 \text{ m})$

$r = 8$ m

Substitute 8 m for **r** in the formula: $V = \frac{4}{3} \pi r^3$

$= \frac{4}{3} \times 3.14 \times (8 \text{ m})^3$

$= \frac{4}{3} \times 3.14 \times 512 \text{ m}^3$

$= 2143.57 \text{ m}^3$

Therefore, the volume of the sphere is **2144 m³** rounded to the nearest cubic meter.

Example 2

Find the volume of a sphere with radius 7.6 m. Use π = 3.14 and round your answer to two decimal places.

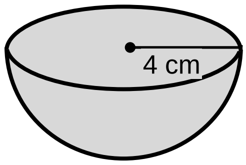
Solution:

$$V = \frac{4}{3}\pi r^3$$
$$V = \frac{4}{3}(3.14)(7.6\text{ m})^3$$
$$V = \frac{4}{3}(3.14)(438.976\text{ m}^3)$$
$$V = \frac{4}{3}(1378.38464\text{ m}^3)$$
$$V = \frac{5513.53856}{3}\text{ m}^3$$
$$V = 1837.84618\text{ m}^3$$
$$V = 1837.85\text{ m}^3 \text{ rounded to two decimal places}$$

Therefore, the volume of the sphere is 1837.85 m^3.

Example 3

Calculate the volume of a hemisphere with a radius of 4 cm. Use π = 3.14 and round your answer to the nearest whole cubic meter.



Solution: As defined, a hemisphere is half sphere, with one circular face and one bowl-shaped face as shown at the right.

Hence, the volume of a hemisphere is equal to half the volume of a sphere.

So, the formula is: $V = \frac{1}{2}(\frac{4}{3}\pi r^3)$

Substitute 4 cm for **r** in the formula:

$$V = \frac{1}{2} \times \frac{4}{3} \times 3.14 \times (4\text{ m})^3$$
$$= \frac{1}{2} \times \frac{4}{3} \times 3.14 \times 64\text{ m}^3$$
$$= \frac{1}{2} \times \frac{4}{3} \times 200.96\text{ m}$$
$$= 133.973\text{ m}^3$$
$$= 134\text{ m}^3 \text{ rounded to the nearest whole cubic meter.}$$

Therefore the volume of the hemisphere is 134 m^3.

We can also change the subject of the formula to obtain the radius of a sphere given the volume.

Example 4

Find the radius of a sphere which has a volume of 1500 m^3. Use π = 3.14. Write the answer correct to three significant figures.

Solution: Given the volume (V) = 1500 m^3. Substitute this in the formula.

$$V = \frac{4}{3}\pi r^3$$

$$1500\ m^3 = \frac{4}{3} \times 3.14 \times r^3$$

$$4500\ m^3 = 12.56 \times r^3$$

$$r^3 = \frac{4500\ cm^3}{12.56}$$

$$r^3 = 358.280\ cm^3$$

$$r = \sqrt[3]{358.280\ cm^3}$$

$$r = 7.102\ cm$$

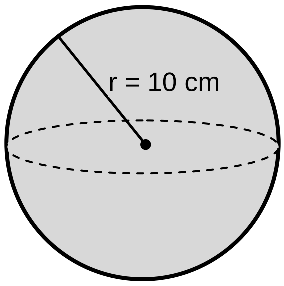
Therefore, r = 7.10 cm correct to 3 significant figures

NOW DO PRACTICE EXERCISE 20

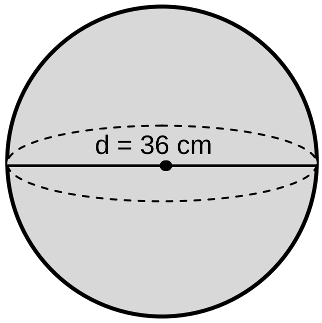
Practice Exercise 20

1. Find the surface area of the following spheres. Take π = 3.14 and give your answers to the nearest tenths. Don't forget to include the relevant unit in your answer.

(a)

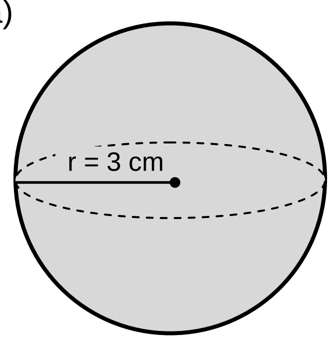


(b)

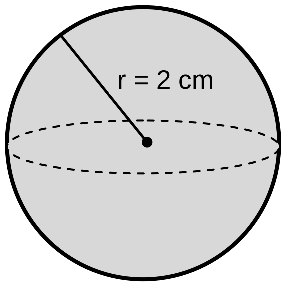


2. Find the volume of the following spheres. Take π = 3.14 and give your answers to the nearest tenths. Don't forget to include the relevant unit in your answer.

(a)



(b)



3. The volume of a bowling ball is 5000 cm^3.

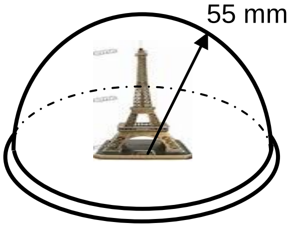
What is the radius of the ball?

---

4. Considering the Earth as a sphere of radius 4×10^4 km, calculate (a) its surface area, (b) its volume. Use $\pi = 3.14$ and give your answer in standard form.

---

5. A paper weight is shaped like a hemisphere with a radius of 55 mm.
What is its volume? Express your answer to the nearest cubic cm.



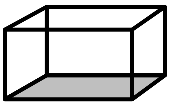
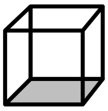
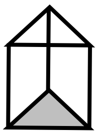
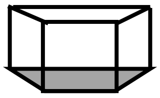
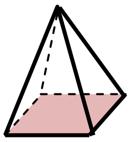
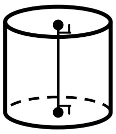
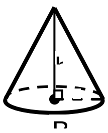
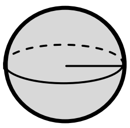
CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4

TOPIC 4: SUMMARY

This summarizes some of the important ideas, concepts and formulas to be remembered.

- The surface area of any solid is equal to the sum of all its lateral faces, including curved ones.
- The volume of any solid is all the space inside the solid.

The table below summarizes the different formulas for surface area and volume of solids.

Name	Shape	TotalSurface Area	Volume
Rectangular Prism		**TSA = 2(LW + HL + HW)**	**V= L X W X H**
Cube		**TSA = $6s^2$**	**V= s^3**
Triangular Prism		**LSA = p x h** **TSA = LSA + 2B**	**V= B x h**
Trapezoidal Prism		**LSA = p x h** **TSA = LSA + 2B**	**V= B x h**
Pyramid		**LSA = $\frac{1}{2}P\ell$** **TSA = B + LSA**	**V= $\frac{1}{3}Bh$**
Cylinder		**LSA = $2\pi r^2$** **TSA = $2\pi r(H + r)$**	**V= $\pi r^2 h$**
Cone		**CSA = $\pi r l$** **TSA = $\pi r(r + l)$**	**V= $\frac{1}{3}\pi r^2 h$**
Sphere		**TSA = $4\pi r^2$**	**V= $\frac{4}{3}\pi r^3$**

REVISE LESSONS 15- 20. THEN DO TOPIC TEST 4 IN ASSIGNMENT BOOK 5.

ANSWERS TO PRACTICE EXERCISES 15 – 20

Practice Exercise 15

1.	(a)	120 cm^2	(b)	684 cm^2	(c)	108 cm^2	(d)	54 cm^2
2.	(a)	12 318 ft^2	(b)	2280 cm^2	(c)	70.73 cm^2	(d)	K950.52

Practice Exercise 16

1.	(a)	12 cm^3	(b)	72 cm^3	(c)	150 cm^3	(d)	330 cm^3
2	(a)	10 m	(b)	52 cm				
3.	(a)	63 cm	(b)	6.05 m				
4.	10 cm							
5.	(a)	38.5 cm^2	(b)	531.3 m^3				

Practice Exercise 17

1. 60 cm^2
2. 384 cm^2
3. 300 cm^3
4. 25 cm^3
5. 64 cm^3

Practice Exercise 18

1.	i)	(a)	628 cm^2	(b)	157 cm^2	(c)	785 cm^2
	ii)	(a)	352 m^2	(b)	1231 m^2	(c)	1583 m^2

2. 251.43 cm^2

3. 376 cm^2

4. (a) 10 x 5 = 50 cm^3
   (b) 2.8 x 2 = 5.6 cm^3
   (c) 305 x 87 = 26 535 cm^3

5. (a) 113.04 cm^2

   (b) 1017.36 cm^3

   (c) The cylinder holds more than1 litre.

Practice Exercise 19

1. 43.96 cm^2
2. 320 cm^2
3. 615.44 cm^3
4. 21π
5. 150.73 cm^2
6. 172.7 cm^2
7. 1206 cm^3

Practice Exercise 20

1. (a) 1256 cm^2 (b) 4069.44 cm^2
2. (a) 113.04 cm^3 (b) 33.49 cm^3
3. 10.9 cm
4. (a) 2.0096×10^{10} km^2 (b) 2.6795×10^{14} km^3
5. 348 cm^3

END OF UNIT 5

REFERENCES

- FODE Grade 9 Formal Mathematics Unit Books 4 and 5 Second Edition
- Stanley Thornes Ltd; **A New General Mathematics Complete Edition** by Ewart Smith
- Pearson Longman, **Signpost Mathematics 8 and 9 Second Edition by** A. McSeveny, R. Conway aand S. Wilkes
- Http://www.mathworkssheet 4kids.com
- Http://www.mathisfun.com/geometry/prism.html
- Http://www.onlinemathlearning .com/surface-area-sphere.html
- Http://www.onlinemathlearning.com/volume-of-a-sphere.html
- Http://www.basic-mathematics.com/volume-of-a-sphere.html
- Http://www.basic-mathematics.com/surface-area-of-a-rectangular-prism.html
- Http://www.onlinemathlearning .com/volume-cone.html
- NDOE **Secondary School Mathematics 9A and 10A**
- Kuta software-infinite pre-Algebra
- Grade 9 mathematics Outcome Edition
- Http://www.mathteacher.com.au/year10/ch14_measurement/18_cone/20con.html
- Shakespeare Head Press, **2 Unit Mathematics Book 1** by S. B. Jones and K.E. Couchman